12^{th} STD

MATHEMATICS

SCHOOL EDUCATION DEPARTMENT

KRISHNAGIRI DISTRICT

HIGHER SECONDARY-SECOND YEAR

MATHEMATICS

SPECIAL GUIDE [2025-26]

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12-ஆம் வகுப்பு

ஒரு மதிப்பெண் வினாக்கள்

12-ஆம் வகுப்பு பாடப்புத்தகத்தில் உள்ள ஒரு மதிப்பெண் வினாக்கள், GeoGebra மென்பொருளின் உதவியோடு,ஒரு வினாவிற்கு சரியான விடையை தேர்வு செய்ய அதிகபட்சம் மூன்று வாய்ப்புகள் வழங்கி, மாணவர்களின் கற்றல் ,கற்பித்தல் திறன் அதிகரிக்கும் வகையில் வடிவமைக்கப்பட்டுள்ளது என்பதை தெரிவித்துக்கொள்கிறோம் .

குறிப்பு : Hi-Tech Lab-ல் QR Code -ஐ Scan செய்து அல்லது Link -ஐ click செய்து மாணவர்கள் பயிற்சி செய்யும் விதமாக மென்பொருள் உருவாக்கப்பட்டுள்ளது .

தமிழ் வழி

https://www.geogebra.org/m/svp4anun

ஆங்கில வழி

https://www.geogebra.org/m/zzajah2u

உருவாக்கம் :

முனைவர்.பொ .ஜெ .முரளி

தலைமை ஆசிரியர்

அரசு மேல்நிலைப்பள்ளி, பாரூர் .

திரு. நா .காளியப்பன்

முதுகலை ஆசிரியர்

அரசு மே.நி.பள்ளி , மோரன அள்ளி.

MATHS

1.APPLICATIONS OF MATRICES AND DETERMINANTS

1. If $adjA = \begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix}$and $adjB = \begin{bmatrix} 1 & -2 \\ -3 & 1 \end{bmatrix}$, then adj (AB) is

   (a) $\begin{bmatrix} -7 & -1 \\ 7 & -9 \end{bmatrix}$ (b) $\begin{bmatrix} -6 & 5 \\ -2 & -10 \end{bmatrix}$ (c) $\begin{bmatrix} -7 & 7 \\ -1 & -9 \end{bmatrix}$ (d) $\begin{bmatrix} -6 & -2 \\ 5 & -10 \end{bmatrix}$

2. The rank of the matrix $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ -1 & -2 & -3 & -4 \end{bmatrix}$ is

   (a) 1 (b) 2 (c) 4 (d) 3

3. If $x^a y^b = e^m, x^c y^d = e^n, \Delta_1 = \begin{vmatrix} m & b \\ n & d \end{vmatrix}, \Delta_2 = \begin{vmatrix} a & m \\ c & n \end{vmatrix}, \Delta_3 = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$ then the values of x and y are respectively,

   (a) $e^{(\Delta_2/\Delta_1)}, e^{(\Delta_3/\Delta_1)}$ (b)$\log(\Delta_1/\Delta_3)$, $\log(\Delta_2/\Delta_3)$ (c) $\log(\Delta_2/\Delta_1)$,$\log(\Delta_3/\Delta_1)$ (d) $e^{(\Delta_1/\Delta_3)}, e^{(\Delta_2/\Delta_3)}$

4. If $A = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$, B=adjA and C=3A , then $\frac{|adjB|}{|C|} =$

   (a) $\frac{1}{3}$ (b) $\frac{1}{9}$ (c) $\frac{1}{4}$ (d) 1

5. If $A\begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$ then A=

   (a) $\begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$ (c) $\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 4 & -1 \\ 2 & 1 \end{bmatrix}$

6. If |adj (adjA)|= $|A|^9$, then the order of the square matrix A is

   (a) 3 (b) 4 (c) 2 (d) 5

7. If A is a 3× 3 non-singular matrix such that $AA^T = A^TA$ and $B = A^{-1}A^T$, then $BB^T =$

   (a) A (b) B (c) I_3 (d) B^T

8. If $A = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$, then $9I_2 - A =$

   (a) A^{-1} (b) $\frac{A^{-1}}{2}$ (c)$3A^{-1}$ (d) $2A^{-1}$

9. If $A = \begin{bmatrix} 2 & 0 \\ 1 & 5 \end{bmatrix}$and $B = \begin{bmatrix} 1 & 4 \\ 2 & 0 \end{bmatrix}$ then $|adj(AB)| =$

   (a) −40 (b) −80 (c) −60 (d) −20

10. If $A = \begin{bmatrix} 3 & 1 & -1 \\ 2 & -2 & 0 \\ 1 & 2 & -1 \end{bmatrix}$ & $A^{-1} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$then the value of a_{23} is

    (a) 0 (b) −2 (c) −3 (d) −1

11. If A, B and C are invertible matrices of some order, then which one of the following is not true?

    (a) adjA=|A |A^{-1} (b) adj(AB)=(adjA)(adjB) (c) detA^{-1} =$(detA)^{-1}$ (d) $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$

12. If $(AB)^{-1} = \begin{bmatrix} 12 & -17 \\ -19 & 27 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}$, then B^{-1}

    (a) $\begin{bmatrix} 2 & -5 \\ -3 & 8 \end{bmatrix}$ (b) $\begin{bmatrix} 8 & 5 \\ 3 & 2 \end{bmatrix}$ (c) $\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 8 & -5 \\ -3 & 2 \end{bmatrix}$

13. If $A^TA^{-1}$is symmetric, then $A^2 =$

    (a) A^{-1} (b) $(A^T)^2$ (c) A^T (d) $(A^{-1})^2$

14. If A is a non-singular matrix such that $A^{-1} = \begin{bmatrix} 5 & 3 \\ -2 & -1 \end{bmatrix}$, then$(A^T)^{-1} =$

    (a) $\begin{bmatrix} -5 & 3 \\ 2 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 5 & 3 \\ -2 & -1 \end{bmatrix}$ (c) $\begin{bmatrix} -1 & -3 \\ 2 & 5 \end{bmatrix}$ (d) $\begin{bmatrix} 5 & -2 \\ 3 & -1 \end{bmatrix}$

15. If $A = \begin{bmatrix} \frac{3}{5} & \frac{4}{5} \\ x & \frac{3}{5} \end{bmatrix}$, and $A^T = A^{-1}$ then the value of x is

    (a) $\frac{-4}{5}$ (b) $\frac{-3}{5}$ (c) $\frac{3}{5}$ (d) $\frac{4}{5}$

16. If $P = \begin{bmatrix} 1 & x & 0 \\ 1 & 3 & 0 \\ 2 & 4 & -2 \end{bmatrix}$ is the adjoint of 3×3 matrix A and |A |=4 ,then x is

(a) 15 (b) 12 (c) 14 (d) 11

17. If $A = \begin{bmatrix} 1 & tan\frac{\theta}{2} \\ -tan\frac{\theta}{2} & 1 \end{bmatrix}$ and $AB = I_2$, then $B =$

(a) $\left(cos^2\frac{\theta}{2}\right)A$ (b) $\left(cos^2\frac{\theta}{2}\right)A^T$ (c) $(cos^2\theta)I$ (d) $\left(sin^2\frac{\theta}{2}\right)A$

18. Which of the following is/are correct?

(i) Adjoint of a symmetric matrix is also a symmetric matrix.

(ii) Adjoint of a diagonal matrix is also a diagonal matrix.

(iii) If A is a square matrix of order n and λ is a scalar, then $adj(\lambda A) = \lambda^n adj(A)$

(iv) A(adjA)= (adjA)A= |A|I

(a) Only (i) (b) (ii) and (iii) (c) (iii) and (iv) (d) (i), (ii) and (iv)

19. If $\rho(A) = \rho[(A|B)]$ then the system $AX = B$ of linear equations is

(a) consistent and has a unique solution (b) consistent (c) consistent and has infinitely many solution (d) inconsistent

20. If $A = \begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix}$ and A(adjA)= $\begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$,then $k =$

(a) 0 (b) sinθ (c) cosθ (d) 1

21. If $0 \le \theta \le \pi$ and the system of equations $x + (sin\theta)y - (cos\theta)z = 0$, $(cos\theta)x - y + z = 0$, $(sin\theta)x + y - z = 0$ has non-trivial solution then θ is

(a) $\frac{2\pi}{3}$ (b) $\frac{3\pi}{4}$ (c) $\frac{5\pi}{6}$ (d) $\frac{\pi}{4}$

22. If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ be such that $\lambda A^{-1} = A$ then λ is

(a) 17 (b) 14 (c) 19 (d) 21

23. If $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$, then adj(adjA) is

(a) $\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 6 & -6 & 8 \\ 4 & -6 & 8 \\ 0 & -2 & 2 \end{bmatrix}$ (c) $\begin{bmatrix} -3 & 3 & -4 \\ -2 & 3 & -4 \\ 0 & 1 & -1 \end{bmatrix}$ (d) $\begin{bmatrix} 3 & -3 & 4 \\ 0 & -1 & 1 \\ 2 & -3 & 4 \end{bmatrix}$

24. The augmented matrix of a system of linear equations is $\begin{bmatrix} 1 & 2 & 7 & 3 \\ 0 & 1 & 4 & 6 \\ 0 & 0 & \lambda-7 & \mu+5 \end{bmatrix}$. The system has infinitely many solutions if

(a) $\lambda = 7, \mu \ne -5$ (b) $\lambda = -7, \mu = 5$ (c) $\lambda \ne 7, \mu \ne -5$ (d) $\lambda = 7, \mu = -5$

25. Let $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and $4B = \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & x \\ -1 & 1 & 3 \end{bmatrix}$. If B is the inverse of A, then the value of x is

(a) 2 (b) 4 (c) 3 (d) 1

2.COMPLEX NUMBERS

1. If $z = \frac{(\sqrt{3}+i)^3(3i+4)^2}{(8+6i)^2}$ then |z | is equal to

(a) 0 (b) 1 (c) 2 (d) 3

2. If z is a non-zero complex number, such that $2iz^2 = \bar{z}$, then |z | is

(a) $\frac{1}{2}$ (b) 1 (c) 2 (d) 3

3. If | z-2+i| ≤ 2 then the greatest value of |z | is

(a) $\sqrt{3} - 2$ (b) $\sqrt{3} + 2$ (c) $\sqrt{5} - 2$ (d) $\sqrt{5} + 2$

4. The solution of the equation |z|-z=1+2i, is

(a) $\frac{3}{2}-2i$ (b) $-\frac{3}{2}+2i$ (c) $2-\frac{3}{2}i$ (d) $2+\frac{3}{2}i$

5. $i^n+i^{n+1}+i^{n+2}+i^{n+3}$ is

(a) 0 (b) 1 (c) −1 (d) i

6. The value of $\sum_{i=1}^{13}(i^n+i^{n-1})$,is

(a) 1+ i (b) i (c) 1 (d) 0

7. The area of the triangle formed by the complex numbers z,iz and z+iz in the Argand's diagram is

(a) $\frac{1}{2}|z|^2$ (b) $|z|^2$ (c) $\frac{3}{2}|z|^2$ (d) $2|z|^2$

8. The conjugate of a complex number is $\frac{1}{i-2}$,Then, the complex number is

(a) $\frac{1}{i+2}$ (b) $\frac{-1}{i+2}$ (c) $\frac{-1}{i-2}$ (d) $\frac{1}{i-2}$

9.If $\omega \neq 1$ is a cubic root of unity and $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -\omega^2-1 & \omega^2 \\ 1 & \omega^2 & \omega^7 \end{vmatrix} = 3k$, then k=

(a) 1 (b) −1 (c) $\sqrt{3}i$ (d) $-\sqrt{3}i$

10. If α and β are the roots of $x^2+x+1=0$ then $\alpha^{2020}+\beta^{2020}$ is

(a) −2 (b) −1 (c) 1 (d) 2

11. The product of all four values of $\left(cos\frac{\pi}{3}+isin\frac{\pi}{3}\right)^{\frac{3}{4}}$ is

(a) −2 (b) −1 (c) 1 (d) 2

12. The value of $\left(\frac{1+\sqrt{3}i}{1-\sqrt{3}i}\right)^{10}$ is

(a) cis$\frac{2\pi}{3}$ (b) cis$\frac{4\pi}{3}$ (c) -cis$\frac{2\pi}{3}$ (d) -cis$\frac{4\pi}{3}$

13. If $\omega = cis\frac{2\pi}{3}$, then the number of distinct roots of $\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix} = 0$

(a) 1 (b) 2 (c) 3 (d) 4

14. If z is a complex number s.t $z \in C\backslash R$ and $z+\frac{1}{z} \in R$, then |z| is

(a) 0 (b) 1 (c) 2 (d) 3

15. z_1, z_2& z_3 be three complex numbers such that $z_1+z_2+z_3=0$ and $|z_1|=|z_2|=|z_3|=1$, then $z_1{}^2+z_2{}^2+z_3{}^2$ is

(a) 3 (b) 2 (c) 1 (d) 0

16. If $\frac{z-1}{z+1}$ is purely imaginary, then |z | is

(a) $\frac{1}{2}$ (b) 1 (c) 2 (d) 3

17. If $\left|z-\frac{3}{z}\right|=2$, then the least value of |z | is

(a) 1 (b) 2 (c) 3 (d) 5

18. If |z|=1 , then the value of $\frac{1+z}{1+\bar{z}}$ is

(a) z (b) $\bar{z}$ (c) $\frac{1}{z}$ (d) 1

19.If $z=x+iy$ is a complex number such that |z+2|=|z-2 |,then the locus of z is

(a) real axis (b) imaginary axis (c) ellipse (d) circle

20. The principal argument of $\frac{3}{-1+i}$ is

(a) $\frac{-5\pi}{6}$ (b) $\frac{-2\pi}{3}$ (c) $\frac{-3\pi}{4}$ (d) $\frac{-\pi}{2}$

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21. The principal argument of $(sin40° + icos40°)^5$ is

(a) -110° (b)-70° (c) 70° (d)110°

22. If $(1+i)(1+2i)(1+3i) \ldots\ldots(1+ni) = (x+iy)$, then $2..5.10 \ldots\ldots(1+n^2)$ is

(a) 1 (b) i (c) $x^2 + y^2$ (d) $1 + n^2$

23. If $\omega \neq 1$is a cubic root of unity and $(1+\omega)^7 = A + B\omega$, then (A, B)=

(a) (1,0) (b) (-1,1) (c) (0,1) (d) (1,1)

24. The principal argument of the complex number $\frac{(1+i\sqrt{3})^2}{4i(1-i\sqrt{3})}$ is

(a) $\frac{2\pi}{3}$ (b) $\frac{\pi}{6}$ (c) $\frac{5\pi}{6}$ (d) $\frac{\pi}{2}$

25. If $|z_1|=1$, $|z_2|=2$, $|z_3|=3$, and $|9z_1z_2 + 4z_1z_3 + z_2z_3|=12$ then the value of $|z_1 + z_2 + z_3|$ is

(a) 1 (b) 2 (c) 3 (d) 4

3.THEORY OF EQUATIONS

1. The polynomial $x^3 + 2x + 3$ has

(a) 1 -ve and 2 imaginary zeros (b) 1 +ve and 2 imaginary zeros

(c) 3 real zeros (d) no zeros

2. The number of +ve roots of the polynomial $\sum_{j=0}^{n} n_{C_r}(-1)^r x^r$ is

(a) 0 (b) n (c) $< n$ (d) r

3. If f and g are polynomials of degrees m and n respectively, and if $h(x) = (f \bullet g)(x)$, then the degree of h is

(a) mn (b)m+n (c) m^n (d) n^m

4. A polynomial equation in x of degree n always has

(a) n distinct roots (b) n real roots (c) n complex roots (d) at most one root.

5. If α , β and γ are the roots of $x^3 + px^2 + qx + r$, then $\Sigma\frac{1}{\alpha}$ is

(a) $\frac{-q}{r}$ (b) $\frac{-p}{r}$ (c) $\frac{q}{r}$ (d) $\frac{-q}{p}$

6. A zero of $x^3 + 64$ is

(a) 0 (b) 4 (c) 4i (d) -4

7. The polynomial $x^3 - kx^2 + 9x$ has three real zeros if and only if, k satisfies

(a) $|k| \leq 6$ (b) $k = 0$ (c) $|k| > 6$ (d) $|k| \geq 6$

8.The number of real numbers $[0,2\pi]$in satisfying $sin^4x - 2sin^2x + 1$ is

(a) 2 (b) 4 (c)1 (d)∞

9. If $x^3 + 12x^2 + 10ax + 1999$ definitely has a positive zero, if and only if

(a) $a \geq 0$ (b) $a > 0$ (c) $a < 0$ (d) $a \leq 0$

10. According to the rational root theorem, which number is not possible rational zero of $4x^7 + 2x^4 - 10x^3 - 5$?

(a) -1 (b)$\frac{5}{4}$ (c) $\frac{4}{5}$ (d) 5

4.INVERSE TRIGNOMETRIC FUNCTIONS

1. If $sin^{-1} x = 2sin^{-1} \alpha$ has a solution, then

(a) $|\alpha| \leq \frac{1}{\sqrt{2}}$ (b) $|\alpha| \geq \frac{1}{\sqrt{2}}$ (c) $|\alpha| < \frac{1}{\sqrt{2}}$ (d) $|\alpha| > \frac{1}{\sqrt{2}}$

2. $sin^{-1}(cosx) = \frac{\pi}{2} - x$ is valid for

(a)$-\pi \leq x \leq 0$ (b) $0 \leq x \leq \pi$ (c) $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$ (d) $-\frac{\pi}{4} \leq x \leq \frac{3\pi}{4}$

3. The value of $sin^{-1}(cosx)$ $0 \leq x \leq \pi$ is

(a) $\pi - x$ (b)$x - \frac{\pi}{2}$ (c) $\frac{\pi}{2} - x$ (d) $x - \pi$

4. If $sin^{-1}x + sin^{-1}y = \frac{2\pi}{3}$ then $cos^{-1}x + cos^{-1}y$ is equal to

(a) $\frac{2\pi}{3}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{6}$ (d) π

5. If $sin^{-1} x + sin^{-1} y + sin^{-1} z = \frac{3\pi}{2}$, the value of $x^{2017} + y^{2018} + z^{2019} - \frac{9}{x^{101}+y^{101}+z^{101}}$ is

(a) 0 (b) 1 (c) 2 (d) 3

6. If $cot^{-1} x = \frac{2\pi}{5}$ for some $\in R$, the value of $tan^{-1} x$ is

(a) $\frac{-\pi}{10}$ (b) $\frac{\pi}{5}$ (c) $\frac{\pi}{10}$ (d) $\frac{-\pi}{5}$

7. The domain of the function defined by $f(x) = sin^{-1}\sqrt{x-1}$ is

(a)[1,2] (b) [-1,1] (c)[0,1] (d)[-1,0]

8. $sin^{-1} \frac{3}{5} - cos^{-1} \frac{12}{13} + sec^{-1} \frac{5}{3} - cosec^{-1} \frac{13}{2}$ is equal to

(a) 2π (b) π (c) 0 (d) $tan^{-1} \frac{12}{65}$

9. If $|x| \le 1$, then $2tan^{-1}x - sin^{-1} \frac{2x}{1+x^2}$ is equal to

(a) $tan^{-1}x$ (b) $sin^{-1}x$ (c) 0 (d) π

10. The equation $tan^{-1}x - cot^{-1}x = tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ has

(a) no solution (b) unique solution (c) two solutions (d) infinite number of solutions

11. If $sin^{-1}x + cot^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{2}$,then x is equal to

(a)$\frac{1}{2}$ (b) $\frac{1}{\sqrt{5}}$ (c) $\frac{2}{\sqrt{5}}$ (d) $\frac{\sqrt{3}}{2}$

12. If $sin^{-1}\frac{x}{5} + cosec^{-1}\frac{5}{4} = \frac{\pi}{2}$, then the value of x is

(a) 4 (b) 5 (c) 2 (d) 3

13. $sin(tan^{-1}x), |x| < 1$ is equal to

(a)$\frac{x}{\sqrt{1-x^2}}$ (b) $\frac{1}{\sqrt{1-x^2}}$ (c) $\frac{1}{\sqrt{1+x^2}}$ (d) $\frac{x}{\sqrt{1+x^2}}$

14. If $x = \frac{1}{5}$, the value of cos $(cos^{-1}x + 2sin^{-1}x)$ is

(a)$-\sqrt{\frac{24}{25}}$ (b) $\sqrt{\frac{24}{25}}$ (c) $\frac{1}{5}$ (d) $\frac{-1}{5}$

15. $tan^{-1}\left(\frac{1}{4}\right) + tan^{-1}\left(\frac{2}{9}\right)$ is equal to

(a) $\frac{1}{2} cos^{-1}\left(\frac{3}{5}\right)$ (b) $\frac{1}{2} sin^{-1}\left(\frac{3}{5}\right)$ (c) $\frac{1}{2} tan^{-1}\left(\frac{3}{5}\right)$ (d) $tan^{-1}\left(\frac{1}{2}\right)$

16. If the function $f(x) = sin^{-1}(x^2-3)$, then x belongs to

(a) [-1,1] (b) $[\sqrt{2}, 2]$ (c) $[-2,-\sqrt{2}] \cup [\sqrt{2},2]$ (d) $[-2,-\sqrt{2}]$

17. If $cot^{-1}2$ and $cot^{-1}3$ are two angles of a triangle, then the 3rd angle is

(a) $\frac{\pi}{4}$ (b) $\frac{3\pi}{4}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{3}$

18. $sin^{-1}\left(tan\frac{\pi}{4}\right) - sin^{-1}\left(\sqrt{\frac{3}{x}}\right) = \frac{\pi}{6}$, Then x is a root of the equation

(a)$x^2 - x - 6 = 0$ (b) $x^2 - x - 12 = 0$ (c) $x^2 + x - 12 = 0$ (d) $x^2 + x - 6 = 0$

19. $sin^{-1}(2cos^2x - 1) + cos^{-1}(1 - 2sin^2x) =$

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

20. If $cot^{-1}(\sqrt{sin\alpha}) + tan^{-1}(\sqrt{sin\alpha}) = u$, then $cos2u$ is equal to

(a) $tan^2\alpha$ (b)0 (c)-1 (d) $tan2\alpha$

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5.TWO DIMENSIONAL ANALYTICAL GEOMETRY

1. If the normal of the parabola $y^2 = 4x$ drawn at the end points of its latus rectum are tangents to the circle $(x-3)^2 + (y+2)^2 = r^2$,then the value of r^2 is

   (a) 2 (b) 3 (c) 1 (d) 4

2. If $x + y = k$ is a normal to the parabola$y^2 = 12x$, then the value of k is

   (a) 3 (b) -1 (c) 1 (d) 9

3. The ellipse $E_1: \frac{x^2}{9} + \frac{y^2}{4} = 1$is inscribed in a rectangle R whose sides are parallel to the coordinate axes. Another ellipse E_2 passing through the point (0,4) circumscribes the rectangle R,. The eccentricity of the ellipse is

   (a) $\frac{\sqrt{2}}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{1}{2}$ (d) $\frac{3}{4}$

4. Tangents are drawn to the hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$ parallel to the straight line $-y = 1$. One of the points of contact of tangents on the hyperbola is

   (a) $\left(\frac{9}{2\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$ (b) $\left(\frac{-9}{2\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ (c) $\left(\frac{9}{2\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ (d) $(3\sqrt{3}, -2\sqrt{2})$

5. The eqn of the circle passing through the foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ having centre at (0,3)is

   (a) $x^2 + y^2 - 6y - 7 = 0$ (b) $x^2 + y^2 - 6y + 7 = 0$ (c) $x^2 + y^2 - 6y - 5 = 0$ (d) $x^2 + y^2 - 6y + 5 = 0$

6. Let C be the circle with centre at (1,1) and radius =1. If T is the circle centred at (0,y) passing through the origin and touching the circle C externally, then the radius of T is equal to

   (a) $\frac{\sqrt{3}}{\sqrt{2}}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

7. Consider an ellipse whose centre is of the origin and its major axis is along x-axis. If its eccentricity is $\frac{3}{5}$ and the distance between its foci is 6, then the area of the quadrilateral inscribed in the ellipse with diagonals as major and minor axis of the ellipse is

   (a) 8 (b) 32 (c) 80 (d) 40

8. Area of the greatest rectangle inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is

   (a) $2ab$ (b) ab (c) $\sqrt{ab}$ (d) $\frac{a}{b}$

9. An ellipse has OB as semi minor axes, F and F' its foci and the angle FBF' is a right angle. Then the eccentricity of the ellipse is

   (a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{1}{\sqrt{3}}$

10. The eccentricity of the ellipse $(x-3)^2 + (y-4)^2 = \frac{y^2}{9}$ is

    (a) $\frac{\sqrt{3}}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{3\sqrt{2}}$ (d) $\frac{1}{\sqrt{3}}$

11. If the two tangents drawn from a point P to the parabola $y^2 = 4x$ are at right angles then the locus of P is

    (a) $2x + 1 = 0$ (b) $x = -1$ (c) $2x - 1 = 0$ (d) $x = 1$

12. The circle passing through (1,-2) and touching the axis of x at (3,0) passing through the point

    (a) (-5, 2) (b) (2,-5) (c) (5, -2) (d) (-2,5)

13. The locus of a point whose distance from (-2,0) is $\frac{2}{3}$ times its distance from the line $x = \frac{-9}{2}$ is

    (a) a parabola (b) a hyperbola (c) an ellipse (d) a circle

14. The values of m for which the line $y = mx + 2\sqrt{5}$ touches the hyperbola $16x^2 - 9y^2 = 144$ are the roots of $x^2 - (a+b)x - 4 = 0$, then the value of (a+b) is

    (a) 2 (b) 4 (c) 0 (d) -2

15. If the coordinates at one end of a diameter of the circle $x^2 + y^2 - 8x - 4y + c = 0$ are (11,2) the coordinates of the other end are

    (a) (-5, 2) (b) (2,-5) (c) (5, -2) (d) (-2,5)

16. The equation of the circle passing through (1,5) and (4,1) and touching y -axis is $x^2 + y^2 - 5x - 6y + 9 + \lambda(4x + 3y - 19) = 0$ where λ is equal to

    (a) $0, \frac{-40}{9}$ (b) 0 (c) $\frac{40}{9}$ (d) $\frac{-40}{9}$

17. The eccentricity of the hyperbola whose latus rectum is 8 and conjugate axis is equal to half the distance between the foci is

    (a) $\frac{4}{3}$ (b) $\frac{4}{\sqrt{3}}$ (c) $\frac{2}{\sqrt{3}}$ (d) $\frac{3}{2}$

18.The circle $x^2+y^2=4x+8y+5$ intersects the line $3x-4y=m$ at two distinct points if

(a) $15<m<65$ (b) $35<m<85$ (c) $-85<m<-35$ (d) $-35<m<15$

19. The length of the diameter of the circle which touches the x -axis at the point (1,0) and passes through the point (2,3).

(a) $\frac{6}{5}$ (b) $\frac{5}{3}$ (c) $\frac{10}{3}$ (d) $\frac{3}{5}$

20. The radius of the circle $3x^2+by^2+4bx-6by+b^2=0$ is

(a) 1 (b) 3 (c) $\sqrt{10}$ (d) $\sqrt{11}$

21. The centre of the circle inscribed in a square formed by the lines $x^2-8x-12=0$ and $y^2-14y+45=0$ is

(a) (4,7) (b) (7,4) (c) (9,4) (d) (4,9)

22. The eqn of the normal to the circle $x^2+y^2-2x-2y+1=0$ which is parallel to the line $2x+4y=3$ is

(a) $x+2y=3$ (b) $x+2y+3=0$ (c) $2x+4y+3=0$ (d) $x-2y+3=0$

23. If $P(x,y)$be any point on $16x^2+25y^2=400$ with foci $F_1(3,0)$ and $F_2(-3,0)$ then PF_1+PF_2 is

(a) 8 (b) 6 (c) 10 (d) 12

24. The radius of the circle passing through the point (6,2) two of whose diameter are $x+y=6$ and $x+2y=4$ is

(a) 10 (b) $2\sqrt{5}$ (c) 6 (d) 4

25. The area of quadrilateral formed with foci of the hyperbolas $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ and $\frac{x^2}{a^2}-\frac{y^2}{b^2}=-1$ is

(a)$4(a^2+b^2)$ (b) $2(a^2+b^2)$ (c) a^2+b^2 (d) $\frac{1}{2}(a^2+b^2)$

6.APPLICTIONS OF VECTOR ALGEBRA

1. If $\vec{a}=\hat{\imath}+\hat{\jmath}+\hat{k}, \vec{b}=\hat{\imath}+\hat{\jmath}$, $\vec{c}=\hat{\imath}$ and $(\vec{a}\times\vec{b})\times\vec{c}=\lambda\vec{a}+\mu\vec{b}$ then the value of $\lambda+\mu$ is

(a) 0 (b) 1 (c) 6 (d) 3

2. If $\vec{a},\vec{b},\vec{c}$ are non-coplanar, non-zero vectors such that $[\vec{a},\vec{b},\vec{c}]=3$ then $\{[\vec{a}\times\vec{b},\vec{b}\times\vec{c},\vec{c}\times\vec{a}]\}^2$ is equal to

(a) 81 (b) 9 (c) 27 (d)18

3. If $\vec{a},\vec{b},\vec{c}$ are three non-coplanar vectors such that $\vec{a}\times(\vec{b}\times\vec{c})=\frac{\vec{b}+\vec{c}}{\sqrt{2}}$ then the angle between $\vec{a}$ and $\vec{b}$ is

(a) $\frac{\pi}{2}$ (b) $\frac{3\pi}{4}$ (c) $\frac{\pi}{4}$ (d) π

4. If the planes $\vec{r}.(2\hat{\imath}-\lambda\hat{\jmath}+\hat{k})=3$ and $\vec{r}.(4\hat{\imath}+\hat{\jmath}-\mu\hat{k})=5$ are parallel, then the value of λ and μ are

(a) $\frac{1}{2},-2$ (b) $\frac{-1}{2},2$ (c) $\frac{-1}{2},-2$ (d) $\frac{1}{2},2$

5.If length of perpendicular from the origin to the plane $2x+3y+\lambda z=1$, λ >0 is $\frac{1}{5}$, then the value of λ is

(a) $2\sqrt{3}$ (b) $3\sqrt{2}$ (c) 0 (d) 1

6. If the volume of the parallelepiped with $\vec{a}\times\vec{b},\vec{b}\times\vec{c},\vec{c}\times\vec{a}$ as coterminous edges is 8 cubic units, then the volume of the parallelepiped with $(\vec{a}\times\vec{b})\times(\vec{b}\times\vec{c})$, $(\vec{b}\times\vec{c})\times(\vec{c}\times\vec{a})$ and $(\vec{c}\times\vec{a})\times(\vec{a}\times\vec{b})$ as coterminous edges is,

(a) 8 cu. units (b) 512 cu. units (c) 64 cu. units (d) 24 cu. Units

7. If $\vec{a}$ and $\vec{b}$ are parallel vectors, then $[\vec{a},\vec{c},\vec{b}]$ is equal to

(a) 2 (b) -1 (c) 1 (d) 0

8. If a vector $\vec{\alpha}$ lies in the plane of $\vec{\beta}$and $\vec{\gamma}$, then

(a) $[\vec{\alpha},\vec{\beta},\vec{\gamma}]=1$ (b) $[\vec{\alpha},\vec{\beta},\vec{\gamma}]=-1$ (c) $[\vec{\alpha},\vec{\beta},\vec{\gamma}]=0$ (d) $[\vec{\alpha},\vec{\beta},\vec{\gamma}]=2$

9. If $\vec{a}.\vec{b}=\vec{b}.\vec{c}=\vec{c}.\vec{a}=0$ then the value of $[\vec{a},\vec{b},\vec{c}]$ is

(a) $|\vec{a}||\vec{b}||\vec{c}|$ (b) $\frac{1}{3}|\vec{a}||\vec{b}||\vec{c}|$ (c) 1 (d) -1

10. If $\vec{a},\vec{b},\vec{c}$ are three unit vectors such that is $\vec{a}$ perpendicular to $\vec{b}$, and is parallel to $\vec{c}$ then $\vec{a}\times(\vec{b}\times\vec{c})$ is equal to

(a) $\vec{a}$ (b) $\vec{b}$ (c) $\vec{c}$ (d) $\vec{0}$

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11. If $[\vec{a},\vec{b},\vec{c}]=1$ then the value of $\frac{\vec{a}.(\vec{b}\times\vec{c})}{(\vec{c}\times\vec{a}).\vec{b}}+\frac{\vec{b}.(\vec{c}\times\vec{a})}{(\vec{a}\times\vec{b}).\vec{c}}+\frac{\vec{c}.(\vec{a}\times\vec{b})}{(\vec{c}\times\vec{b}).\vec{a}}$ is

(a) 1 (b) -1 (c) 2 (d) 3

12. The volume of the parallelepiped with its edges represented by the vectors $\hat{\imath}+\hat{\jmath},\ \hat{\imath}+2\hat{\jmath},\ \hat{\imath}+\hat{\jmath}+\pi\hat{k}$ is

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) π (d) $\frac{\pi}{4}$

13. If $\vec{a}$ and $\vec{b}$ are unit vectors such that $[\vec{a},\vec{b},\vec{a}\times\vec{b}]=\frac{\pi}{4}$, then the angle between $\vec{a}$ and $\vec{b}$ is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

14. Consider the vectors $\vec{a},\vec{b},\vec{c},\vec{d}$ such that $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})=\vec{0}$. Let P_1 and P_2 be the planes determined by the pairs of vectors $\vec{a},\vec{b}$ and $\vec{c},\vec{d}$ respectively. Then the angle between P_1 and P_2 is

(a) 0° (b) 45° (c) 60° (d) 90°

15. If $\vec{a}\times(\vec{b}\times\vec{c})=(\vec{a}\times\vec{b})\times\vec{c}$ where $\vec{a},\vec{b},\vec{c}$ are any three vectors such that $\vec{b}.\vec{c}\neq 0$ and $\vec{a}.\vec{b}\neq 0$ then $\vec{a}$ and $\vec{c}$ are

(a) perpendicular (b) parallel (c) inclined at $\frac{\pi}{3}$ (d) inclined at $\frac{\pi}{6}$

16. If $\vec{a}=2\hat{\imath}+3\hat{\jmath}-\hat{k}$, $\vec{b}=\hat{\imath}+2\hat{\jmath}-5\hat{k}$ and $\vec{c}=3\hat{\imath}+5\hat{\jmath}-\hat{k}$ then a vector perpendicular to $\vec{a}$ and lies in the plane containing $\vec{b}$ and $\vec{c}$ is

(a) $-17\hat{\imath}+21\hat{\jmath}-97\hat{k}$ (b) $17\hat{\imath}+21\hat{\jmath}-123\hat{k}$ (c) $-17\hat{\imath}-21\hat{\jmath}+97\hat{k}$ (d) $-17\hat{\imath}-21\hat{\jmath}-97\hat{k}$

17. The angle between the lines $\frac{x-2}{3}=\frac{y+1}{-2}, z=2$ and $\frac{x-1}{1}=\frac{2y+3}{3}=\frac{z+5}{2}$ is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

18. If the line $\frac{x-2}{3}=\frac{y-1}{-5}=\frac{z+2}{2}$ lies in the plane $x+3y-\alpha z+\beta=0$ then (α,β) is

(a) (-5,5) (b) (-6, 7) (c) (5, -5) (d) (6, -7)

19. The angle between the line $\vec{r}=(\hat{\imath}+2\hat{\jmath}-3\hat{k})+t(2\hat{\imath}+\hat{\jmath}-2\hat{k})$ and the plane $\vec{r}.(\hat{\imath}+\hat{\jmath})+4=0$ is

(a) 0° (b) 30° (c) 45° (d) 90°

20. The coordinates of the point where the line $\vec{r}=(6\hat{\imath}-\hat{\jmath}-3\hat{k})+t(-\hat{\imath}+4\hat{k})$ meets the plane $\vec{r}.(\hat{\imath}+\hat{\jmath}-\hat{k})=3$ are

(a) (2, 1, 0) (b) (7, -1, -7) (c) (1, 2, -6) (d) (5, -1, 1)

21. Distance from the origin to the plane $3x-6y+2z+7=0$ is

(a) 0 (b) 1 (c) 2 (d) 3

22. The distance between the planes $x+2y+3z+7=0$ and $2x+4y+6z+7=0$ is

(a) $\frac{\sqrt{7}}{2\sqrt{2}}$ (b) $\frac{7}{2}$ (c) $\frac{\sqrt{7}}{2}$ (d) $\frac{7}{2\sqrt{2}}$

23. If the direction cosines of a line are $\frac{1}{c},\frac{1}{c},\frac{1}{c}$,then

(a) $c=\pm 3$ (b) $c=\pm\sqrt{3}$ (c) c > 0 (d) 0 < c < 1

24. The vector equation $\vec{r}=(\hat{\imath}-2\hat{\jmath}-\hat{k})+t(6\hat{\imath}-\hat{k})$ represents a straight line passing through the points

(a) (0,6,-1) and (1,-2,-1) (b) (0,6,-1) and (1,-4,-2) (c) (1,-2,-1) and (1,4,-2) (d) (1,-2,-1) and (0,-6,1)

25. If the distance of the point (1,1,1) from the origin is half of its distance from the plane $x+y+z+k=0$, then the values of k are

(a) ± 3 (b) ± 6 (c) -3, 9 (d) 3, -9

7.APPLICATIONS OF DIFFERENTIAL CALCULUS

1. The minimum value of the function $|3-x|+9$ is

(a) 0 (b) 3 (c) 6 (d) 9

2. The max slope of the tangent to the curve $y=e^x \sin nx, x\in[0,2\pi]$ is at

(a) $x=\frac{\pi}{4}$ (b) $x=\frac{\pi}{2}$ (c) $x=\pi$ (d) $x=\frac{3\pi}{2}$

3.The maximum value of the function $x^2e^{-2x}, x > 0$ is

(a) $\frac{1}{e}$ (b) $\frac{1}{2e}$ (c) $\frac{1}{e^2}$ (d) $\frac{4}{e^4}$

4.One of the closest point on the curve $x^2 - y^2 = 4$ to the point (6,0) is

(a) (2,0) (b) $(\sqrt{5}, 1)$ (c) $(3, \sqrt{5})$ (d)$(\sqrt{13}, -\sqrt{3})$

5.The maximum value of the product of two positive numbers, when the sum of squares is 200, is

(a) 100 (b) $25\sqrt{7}$ (c) 28 (d) $24\sqrt{14}$

6.The curve $y = ax^4 + bx^2$ with ab>0

(a)has no horizontal tangent (b) is concave up (c)is concave down (d)has no point of inflection

7.The point of inflection of the curve $y = (x-1)^3$ is

(a) (0,0) (b) (0,1) (c) (1,0) (d) (1,1)

8.The volume of sphere is increasing in volume at the rate of $3\pi\ cm^3/sec$. the rate of change of its radius is $\frac{1}{2}\ cm$

(a) $3cm/s$ (b) 2 cm/s (c) 1 cm/s (d) $\frac{1}{2} cm/s$

9.A balloon rises straight up 10 m/s. an observer is 40m away from the spot where the balloon left the ground. Find the rate of change of the balloon's angle of elevation in radian per second when the balloon is 30metres above the ground.

(a) $\frac{3}{25} radians/sec$ (b) $\frac{4}{25} radians/sec$ (c) $\frac{1}{5} radians/sec$ (d) $\frac{1}{3} radians/sec$

10.The position of the particle moving along a horizontal line of any time t is given by $s(t) = 3t^2 - 2t - 8$ .the time at which the particle is at rest is

(a) $t = 0$ (b) $t = \frac{1}{3}$ (c) $t = 1$ (d) $t = 3$

11.A stone is thrown up vertically. The height it reaches at time t seconds is given by $x = 80t - 16t^2$ the stone reaches the max height in t seconds is given by

(a) 2 (b) 2.5 (c) 3 (d)3.5

12.Find the point on the curve $6y = x^3 + 2$ at which y-coordinate change 8 times as fast as x- coordinate is

(a) (4,11) (b) (4,-11) (c) (-4,11) (d) (-4,-11)

13.The value of the limit $\lim_{x\to 0}\left(cotx - \frac{1}{x}\right)$ is

(a)0 (b)1 (c) 2 (d)∞

14.The tangent to the curve $y^2 - xy + 9 = 0$ is vertical when

(a) $y = 0$ (b) $y = \pm\sqrt{3}$ (c) $y = \frac{1}{2}$ (d) $y = \pm 3$

15.Angle between $y^2 = x$ and $x^2 = y$ at the origin is

(a)$tan^{-1}\frac{3}{4}$ (b) $tan^{-1}\left(\frac{4}{3}\right)$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

16.The function $sin^4x + cos^4x$ is increasing in the interval

(a)$\left[\frac{5\pi}{8}, \frac{3\pi}{4}\right]$ (b) $\left[\frac{\pi}{2}, \frac{5\pi}{8}\right]$ (c) $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$ (d) $\left[0, \frac{\pi}{4}\right]$

17.The no. given by Rolle's theorem for the function $x^3 - 3x^2, x \in [0,3]$is

(a) 1 (b) $\sqrt{2}$ (c) $\frac{3}{2}$ (d)2

18.The abscissa of the point on the curve $f(x) = \sqrt{8-2x}$ at which the slope of the tangent is -0.25?

(a) -8 (b) -4 (c) -2 (d)0

19.The slope of the normal line of the curve $f(x) = 2cos4x$ at $x = \frac{\pi}{12}$ is

(a) $-4\sqrt{3}$ (b) -4 (c) $\frac{\sqrt{3}}{12}$ (d) $4\sqrt{3}$

20. The number given by the Mean value theorem for the function $\frac{1}{x}, x \in [1,9]$ is

(a) 2 (b) 2.5 (c) 3 (d)3.5

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8.DIFFERENTIALS AND PARTIAL DERIVATIVES

1.If $g(x,y) = 3x^2 - 5y + 2y^2, x(t) = e^t$ and $y(t) = cost$,then $\frac{\partial g}{\partial t} =$

(a) $6e^{2t} + 5sint - 4costsint$ (b) $6e^{2t} - 5sint + 4costsint$

(c) $3e^{2t} + 5sint + 4costsint$ (d) $3e^{2t} - 5sint + 4costsint$

2.If $f(x) = \frac{x}{x+1}$,then its differential is given by

(a) $\frac{-1}{(x+1)^2}dx$ (b) $\frac{1}{(x+1)^2}dx$ (c) $\frac{1}{x+1}dx$ (d) $\frac{-1}{x+1}dx$

3.If $u(x,y) = x^2 + 3xy + y - 2019$,then $\frac{\partial u}{\partial x}\Big|_{(4,-5)}$ is equal to

(a) -4 (b) -3 (c)-7 (d) 13

4. A circular template has a radius of 10 cm. The measurement of radius has an approximate error of 0.02 cm. then the percentage error in calculating area of this template is

(a) 0.2% (b) 0.4% (c) 0.04% (d) 0.08%

5. The percentage error of fifth root of 31is approximately how many times the percentage error in 31?

(a) $\frac{1}{31}$ (b) $\frac{1}{5}$ (c) 5 (d)31

6. If $(x,y) = e^{x^2+y^2}$, then $\frac{\partial u}{\partial x}$ is equal to

(a) $e^{x^2+y^2}$ (b) $2xu$ (c) x^2u (d) y^2u

7. If $v(x,y) = log(e^x + e^y)$,then $\frac{\partial v}{\partial x} + \frac{\partial v}{\partial y}$ is equal to

(a) $e^x + e^y$ (b) $\frac{1}{e^x+e^y}$ (c) 2 (d)1

8.The change in the surface area $S = 6x^2$ of a cube when the edge length varies from x_0 to $x_0 + dx$ is

(a) $12x_0 + dx$ (b) $12x_0dx$ (c) $6x_0dx$ (d) $6x_0 + dx$

9.If $f(x,y,z) = xy + yz + zx$, then $f_x - f_z$ is equal to

(a) $z - x$ (b) $y - z$ (c) $x - z$ (d) $y - x$

10.The approximate change in the volume V of a cube of side x metres caused by increasing the side by 1% is

(a) $0.3xdxm^3$ (b) $0.03xm^3$ (c) $0.03x^2m^3$ (d) $0.03x^3m^3$

11.Linear approximation for $g(x) = cosx$ at $x = \frac{\pi}{2}$ is

(a) $x + \frac{\pi}{2}$ (b) $-x + \frac{\pi}{2}$ (c) $x - \frac{\pi}{2}$ (d) $-x - \frac{\pi}{2}$

12.If $w(x,y,z) = x^2(y-z) + y^2(z-x) + z^2(x-y)$, then $\frac{\partial w}{\partial x} + \frac{\partial w}{\partial y} + \frac{\partial w}{\partial z}$ is

(a)$xy + yz + zx$ (b) $x(y+z)$ (c) $y(z+x)$ (d) 0

13. If $w(x,y) = x^y, x > 0$, then $\frac{\partial w}{\partial x}$ is equal to

(a) $x^y \log x$ (b) $y\log x$ (c) yx^{y-1} (d) $x \log y$

14. If $f(x,y) = e^{xy}$,then $\frac{\partial^2 f}{\partial x \partial y}$ is equal to

(a) xye^{xy} (b) $(1+xy)e^{xy}$ (c) $(1+y)e^{xy}$ (d) $(1+x)e^{xy}$

15. If we measure the side of the cube to be 4 cm with an error of 0.1 cm, then the error in our calculation of the volume is

(a) 0.4 $cu.cm$ (b) 0.45 $cu.cm$ (c) 2 $cu.cm$ (d) 4.8 $cu.cm$

9.APPLICATIONS OF INTEGRATION

1.The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} sin^2xcosxdx$ is

(a) $\frac{3}{2}$ (b) $\frac{1}{2}$ (c) 0 (d) $\frac{2}{3}$

2.The value of $\int_{-4}^{4}\left[tan^{-1}\left(\frac{x^2}{x^4+1}\right)+tan^{-1}\left(\frac{x^4+1}{x^2}\right)\right]dx$

(a) π (b) 2π (c) 3π (d) 4π

3.The value of $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\left(\frac{2x^7-3x^5+7x^3-x+1}{cos^2x}\right)dx$ is

(a) 4 (b) 3 (c) 2 (d) 0

4.The value of $\int_0^{\frac{2}{3}}\frac{dx}{\sqrt{4-9x^2}}$ is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{4}$ (d) π

5.For any value of $n\in\mathbb{Z}$, $\int_0^{\pi}e^{cos^2x}cos^3x[(2n+1)x]dx$ is

(a) $\frac{\pi}{2}$ (b) π (c) 0 (d) 2

6.The value of $\int_{-1}^{2}|x|dx$ is

(a) $\frac{1}{2}$ (b) $\frac{3}{2}$ (c) $\frac{5}{2}$ (d) $\frac{7}{2}$

7.If $f(x)=\int_0^x tcostdt$, then $\frac{df}{dx}=$

(a) $cosx-xsinx$ (b) $sinx+xcosx$ (c) $xcosx$ (d) $xsinx$

8. The area between $y^2=4x$ and its latus rectum is

(a) $\frac{2}{3}$ (b) $\frac{4}{3}$ (c) $\frac{8}{3}$ (d) $\frac{5}{3}$

9.The value of $\int_0^1 x(1-x)^{99}dx$ is

(a) $\frac{1}{11000}$ (b) $\frac{1}{10100}$ (c) $\frac{1}{10010}$ (d) $\frac{1}{10001}$

10. The value of $\int_0^{\pi}\frac{dx}{1+5^{cosx}}$ is

(a) $\frac{\pi}{2}$ (b) π (c) $\frac{3\pi}{2}$ (d) 2π

11.If $\int_0^x f(t)dt=x+\int_x^1 tf(t)dt$, then the value of $f(1)$ is

(a) $\frac{1}{2}$ (b) 2 (c) 1 (d) $\frac{3}{4}$

12.If $\frac{\Gamma(n+2)}{\Gamma(n)}=90$ then n is

(a) 10 (b) 5 (c) 8 (d) 9

13.the value of $\int_0^{\frac{\pi}{6}}cos^3 3xdx$ is

(a) $\frac{2}{3}$ (b) $\frac{2}{9}$ (c) $\frac{1}{9}$ (d) $\frac{1}{3}$

14. The value of $\int_0^1(sin^{-1}x)^2dx$ is

(a) $\frac{\pi^2}{4}-1$ (b) $\frac{\pi^2}{4}+2$ (c) $\frac{\pi^2}{4}+1$ (d) $\frac{\pi^2}{4}-2$

Z5. The value of $\int_0^{\infty}e^{-3x}x^2dx$ is

(a) $\frac{7}{27}$ (b) $\frac{5}{27}$ (c) $\frac{4}{27}$ (d) $\frac{2}{27}$

16. the value of $\int_0^a\left(\sqrt{a^2-x^2}\right)^3dx$ is

(a) $\frac{\pi a^3}{16}$ (b) $\frac{3\pi a^4}{16}$ (c) $\frac{3\pi a^2}{8}$ (d) $\frac{3\pi a^4}{8}$

17. The value of $\int_0^{\pi}sin^4xdx$ is

(a) $\frac{3\pi}{10}$ (b) $\frac{3\pi}{8}$ (c) $\frac{3\pi}{4}$ (d) $\frac{3\pi}{2}$

18.If $\int_0^a \frac{1}{4+x^2} dx = \frac{\pi}{8}$ then a is

(a)4 (b) 1 (c) 3 (d)2

19.the volume of solid of revolution of the region bounded by $y^2 = x(a-x)$ about x-axis is

(a) πa^3 (b) $\frac{\pi a^3}{4}$ (c) $\frac{\pi a^3}{5}$ (d) $\frac{\pi a^3}{6}$

20.If $f(x) = \int_1^x \frac{e^{sinu}}{u} du, x > 1$ and $\int_1^3 \frac{e^{sinx^2}}{x} dx = \frac{1}{2}[f(a) - f(1)]$, then one of the value of a is

(a) 3 (b) 6 (c) 9 (d)5

10.ORIDNARY DIFFERENTIAL EQUATIONS

1.Integrating factor of the differential equation $\frac{dy}{dx} = \frac{x+y+1}{x+1}$

(a) $\frac{1}{x+1}$ (b) $x+1$ (c) $\frac{1}{\sqrt{x+1}}$ (d) $\sqrt{x+1}$

2.The population P in any year t is such that the rate of increase in the population is proportional to the population. Then

(a) $P = ce^{kt}$ (b) $P = ce^{-k}$ (c) $P = ckt$ (d) $P = c$

3. P is the amount of certain substance left in after time t. If the rate of evaporation of the substance is proportional to the amount of remaining, then

(a) $P = ce^{kt}$ (b) $P = ce^{-kt}$ (c) $P = ckt$ (d) $Pt = c$

4.If the solution of the differential equation $\frac{dy}{dx} = \frac{ax+3}{2y+f}$ represents a circle ,then the value of a is

(a) 2 (b) -2 (c) 1 (d)-1

5.The slope at any point of a curve $y = f(x)$ is given by $\frac{dy}{dx} = 3x^2$ and it passes through (-1,1). Then the equation of the curve is

(a) $y = x^3 + 2$ (b) $y = 3x^2 + 4$ (c) $y = 3x^3 + 4$ (d) $y = x^3 + 5$

6.The order and degree of the differential eqn $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^{1/3} + x^{1/4} = 0$ are respectively

(a) 2,3 (b) 3,3 (c) 2,6 (d) 2,4

7.The differential eqn representing the family of curves $y = Acos(x+B)$ where A and B are parameters, is

(a) $\frac{d^2y}{dx^2} - y = 0$ (b) $\frac{d^2y}{dx^2} + y = 0$ (c) $\frac{d^2y}{dx^2}$ =0 (d) $\frac{d^2x}{dy^2} = 0$

8. The order and degree of the D.E $\sqrt{sinx}(dx + dy) = \sqrt{cosx}(dx - dy)$ is

(a) 1,2 (b) 2,2 (c) 1,1 (d)2,1

9. The order of the differential equation of all circles with centre at (h, k) and radius 'a'is

(a) 2 (b)3 (c) 4 (d)1

10. The differential equation of the family of curves $y = Ae^x + Be^{-x}$ where A and B are arbitrary constants is

(a) $\frac{d^2y}{dx^2} + y = 0$ (b) $\frac{d^2y}{dx^2} - y = 0$ (c) $\frac{dy}{dx} + y = 0$ (d) $\frac{dy}{dx} - y = 0$

11.Tthe general solution of the differential equation $\frac{dy}{dx} = \frac{y}{x}$ is

(a) $xy = k$ (b) $y = klogx$ (c) $y = kx$ (d) $logy = kx$

12.The solution of the differential equation $\frac{dy}{dx} = 2xy$

(a)$y = ce^{x^2}$ (b)$y = 2x^2 + c$ (c) $y = ce^{-x^2} + c$ (d) $y = x^2 + c$

13.The general solution of the differential equation $log\left(\frac{dy}{dx}\right) = (x+y)$

(a) $e^x + e^y = c$ (b) $e^x + e^{-y} = c$ (c) $e^{-x} + e^y = c$ (d) $e^{-x} + e^{-y} = c$

14.The solution of $\frac{dy}{dx} = 2^{y-x}$

(a) $2^x + 2^y = c$ (b) $2^x - 2^y = c$ (c) $\frac{1}{2^x} - \frac{1}{2^y} = c$ (d)$x = y = c$

15.The solution of the differential equation $\frac{dy}{dx}=\frac{y}{x}+\frac{\emptyset\left(\frac{y}{x}\right)}{\emptyset'\left(\frac{y}{x}\right)}$ is

(a) $x\emptyset\left(\frac{y}{x}\right)=k$ (b) $\emptyset\left(\frac{y}{x}\right)=kx$ (c) $y\emptyset\left(\frac{y}{x}\right)=k$ (d) $\emptyset\left(\frac{y}{x}\right)=ky$

16.If $sinx$ is the integrating factor of linear differential eqn $\frac{dy}{dx}+Py=Q$

(a) $logsinx$ (b) $cosx$ (c) $tanx$ (d) $cotx$

17.The number of arbitrary constants in the general solution of order n and $n+1$ respectively

(a) $n-1, n$ (b) $n, n+1$ (c) $n+1, n+2$ (d) $n+1, n$

18.The solution of the differential equation $2x\frac{dy}{dx}-y=3$ represents

(a) straight line (b) circles (c)parabola (d) ellipse

19.The solution of $\frac{dy}{dx}+p(x)y=0$ is

(a) $y=ce^{\int pdx}$ (b) $y=ce^{-\int pdx}$ (c) $x=ce^{-\int pdy}$ (d) $x=ce^{\int pdy}$

20.The integrating factor of the differential equation $\frac{dy}{dx}+y=\frac{1+y}{x}$ is

(a) $\frac{x}{e^{\lambda}}$ (b) $\frac{e^{\lambda}}{x}$ (c) λe^{x} (d) e^{x}

21.The integrating factor of the differential eqn $\frac{dy}{dx}+p(x)y=Q(x)$ is x, then $p(x)$ is

(a) x (b) $\frac{x^2}{2}$ (c) $\frac{1}{x}$ (d) $\frac{1}{x^2}$

22.The deg of the differential eqn $y(x)=1+\frac{dy}{dx}+\frac{1}{1.2}\left(\frac{dy}{dx}\right)^2+\frac{1}{1.2.3}\left(\frac{dy}{dx}\right)^3+\cdots$ is

(a)2 (b) 3 (c) 1 (d) 4

23.If p and q are the order and degree of the differential equation $y\frac{dy}{dx}+x^3\left(\frac{d^2y}{dx^2}\right)+xy=cosx$, when

(a) p < q (b)p = q (c) p > q (d) p exist and q does not exist

24.The solution of the differential equation $\frac{dy}{dx}+\frac{1}{\sqrt{1-x^2}}=0$ is

(a) $y+sin^{-1}x=c$ (b) $x+sin^{-1}y=c$ (c) $y^2+2sin^{-1}x=c$ (d) $x^2+2sin^{-1}y=c$

25.The number of arbitrary constants in the particular solution of the differential equation of third order is

(a) 3 (b) 2 (c) 1 (d)0

11.PROBABILITY DISTRIBUTION

1. Let x have a Bernoulli distribution with mean 0.4 then variance of (2X-3) is

(a)0.24 (b)0.48 (c) 0.6 (d) 0.96

2. If in 6 trials, X is a binomial variable which follows the relation 9P(X=4)=P(X=2)then the probability of success is

(a)0.125 (b)0.25 (c)0.375 (d)0.75

3. A computer salesperson knows from his past experience that he sells computers to one in every twenty customers who enter the showroom. What is the probability that he will sell a computer to exactly two of the next three customers?

(a) $\frac{57}{20^3}$ (b) $\frac{57}{20^2}$ (c) $\frac{19^3}{20^3}$ (d) $\frac{57}{20}$

4.Let X be a random variable with probability density function $f(x)=\begin{cases}\frac{2}{x^3}, & x\geq 1\\ 0, & x<1\end{cases}$ which of the following statement is correct?

(a) both mean and variance exist (b) mean exist but variance does not exist

(c) both mean and variance do not exist (d)variance exist but mean does not exist

MATHS

5.A rod of length $2l$ is broken into two pieces at random. The probability density function of the shorter of the two pieces is $f(x) = \begin{cases} \frac{1}{l}, & 0 < x < l \\ 0, & l \leq x < 2l \end{cases}$ The mean and variance of the shorter of the two pieces are respectively

(a) $\frac{l}{2}, \frac{l^2}{3}$ (b) $\frac{l}{2}, \frac{l^2}{6}$ (c) $l, \frac{l^2}{12}$ (d) $\frac{l}{2}, \frac{l^2}{12}$

6.Consider a game where the player tosses a six sided fair die. If the face that comes up is 6, the player wins Rs.36, otherwise he loses Rs. k^2,where k is the face that comes up $k = \{1,2,3,4,5\}$.

The expected amount to win at this game in Rs. is

(a) $\frac{19}{6}$ (b) $\frac{-1}{6}$ (c) $\frac{3}{2}$ (d) $\frac{-3}{2}$

7. Four buses carrying 160 students from the same school arrive at a football stadium. The buses carry,respectively,42,36,34 and 48 students.one of the students is randomly selected. Let X denote the number of students that were on the bus carrying the randomly selected student.one of the 4 bus drivers is also randomly selected. Let Y denote the number of students on the bus. Then E[X] and E[Y] respectively are

(a) 50 ,40 (b) 40,50 (c) 40.75,40 (d) 41,41

8.If $P(X = 0) = 1 - P(X = 1)$. If $E[X] = 3Var(X)$, then $P(X = 0)$ is

(a) $\frac{2}{3}$ (b) $\frac{2}{5}$ (c) $\frac{1}{5}$ (d) $\frac{1}{3}$

9.A pair of dice numbered 1,2,3,4,5,6 of a six-sided die and 1,2,3,4 of a four sided die is rolled and the sum is determined. Let the random variable X denote this sum. Then the number if elements in the inverse image of 7 is

(a) 1 (b) 2 (c) 3 (d)4

10.A random variable X has binomial distribution with n=25 and p=0.8 then standard deviation of X is

(a) 6 (b) 4 (c) 3 (d)2

11.Let X represent the difference between the number of heads and the number of tails obtained when a coin is tossed n times. Then the possible value of X are

(a) $i + 2n, i = 0,1,2, \ldots n$ (b) $2i - n, i = 0,1,2, \ldots n$ (c) $n - i, i = 0,1,2, \ldots n$ (d) $2i + 2n, i = 0,1,2, \ldots n$

12.If the function $f(x) = \frac{1}{12} for\ a < x < b$ represents a probability density function of a continuous random variable X, then which of the following cannot be the value of a and b?

(a) 0 and 12 (b) 5 and 17 (c) 7 and 19 (d) 16 and 24

13. If X is a bionomial random variable with expected value 6 and variance 2.4 then $P(X = 5)$ is

(a) $\binom{10}{5}\left(\frac{3}{5}\right)^6\left(\frac{2}{5}\right)^4$ (b) $\binom{10}{5}\left(\frac{3}{5}\right)^{10}$ (c) $\binom{10}{5}\left(\frac{3}{5}\right)^4\left(\frac{2}{5}\right)^6$ (d) $\binom{10}{5}\left(\frac{3}{5}\right)^5\left(\frac{2}{5}\right)^5$

14.Two coins are to be flipped. The first coin will land on heads with probability 0.6, the second with probability 0.5. assume that the results of the flips are independent, And let X equal the total number of heads that result. The value of E[X] is

(a) 0.11 (b) 1.1 (c) 11 (d)1

15. On a multiple choice exam with 3 possible destructives for each of the 5 questions, the probability that a student will get 4 or more correct answers just by guessing is

(a) $\frac{11}{243}$ (b) $\frac{3}{8}$ (c) $\frac{1}{243}$ (d) $\frac{5}{243}$

16. The random variable X has the probability density function $f(x) = \begin{cases} ax + b, & 0 < x < 1 \\ 0, & otherwise \end{cases}$ and $E[X]=\frac{7}{12}$, then a and b respectively

(a) 1 and $\frac{1}{2}$ (b) $\frac{1}{2}$ and 1 (c) 2 and 1 (d)1 and 2

17. Suppose that X takes on one of the values 0, 1 and 2.If for some constant k ,$P(X = i) = kP(X = i - 1)$ for $i = 1,2$ and $P(X = 0) = \frac{1}{7}$ then the value of k is

(a) 1 (b) 2 (c) 3 (d)4

18. Which of the following is a discrete random variable?

I. The number of cars crossing a particular signal in a day.

II. The number of customers in a queue to buy train tickets at a moment .

III. The time taken to complete a telephone call.

(a) I and II (b) II only (c)III only (d) II and III

19. If $f(x) = \begin{cases} 2x, & 0 \le x \le a \\ 0, & otherwise \end{cases}$ is a probability density function of a random variable , then the value of a is

(a) 1 (b) 2 (c) 3 (d)4

20. The probability mass function of a random variable is defined as

x	-2	-1	0	1	2
$f(x)$	k	2k	3k	4k	5k

Then E[x] is equal to

(a) $\frac{1}{15}$ (b) $\frac{1}{10}$ (c) $\frac{1}{3}$ (d) $\frac{2}{3}$

12.DISCRETE MATHEMATICS

1. The operation $*$ defined by $a * b = \frac{ab}{7}$ is not a binary operation on

(a) Q^+ (b) z (c) R (d) C

2. In the set Q defined $a \odot b = a + b + ab$ for what value of y, $3 \odot (y \odot 5) = 7$?

(a) $y = \frac{2}{3}$ (b) $y = \frac{-2}{3}$ (c) $y = \frac{-3}{2}$ (d) $y = 4$

3. If $a * b = \sqrt{a^2 + b^2}$ on a real numbers then $*$ is

(a) commutative but not associative (b) associative but not commutative

(c) both commutative and associative (d) neither commutative nor associative

4. Which one of the following statements has the truth value T?

(a) $sinx$ is an even function (b) Every square matrix is non-singular

(c) The product of complex number and its conjugate is purly imaginary (d)$\sqrt{5}$ is an irrational number

5. Which one of the following statements has the truth value F?

(a) Chennai is in india or $\sqrt{2}$ is an integer (b) Chennai is in india or $\sqrt{2}$ is an irrational number

(c) Chennai is in china or $\sqrt{2}$ is an integer (d) Chennai is in china or $\sqrt{2}$ is an irrational number

6. A binary operation on a set S is a function from

(a) $S \to S$ (b) $(S \times S) \to S$ (c) $S \to (S \times S)$ (d) $(S \times S) \to (S \times S)$

7. Subtraction is not a binary operation in

(a) R (b) Z (c) N (d) Q

8. Which one is the contrapositive of the statement $(p \vee q) \to r$?

(a) $\neg r \to (\neg p \wedge \neg q)$ (b) $\neg r \to (p \vee q)$ (c) $r \to (p \wedge q)$ (d) $p \to (q \vee r)$

9. The truth table for $(p \wedge q) \vee \neg q$ is given, which one of the following is true?

p	q	$(p \wedge q) \vee (\neg q)$
T	T	(a)
T	F	(b)
F	T	(c)
F	F	(d)

	(1)	(2)	(3)	(4)
(a)	T	T	T	T
(b)	T	F	T	T
(c)	T	T	F	T
(d)	T	F	F	F

10. In the last column of the truth table for $\neg(p \vee \neg q)$ the number of final outcomes of the truth value 'F' are

(a) 1 (b) 2 (c) 3 (d) 4

11. Which one of the following is incorrect? For any two propositions p and q , we have

(a) $\neg(p \vee q) \equiv \neg p \wedge \neg q$ (b) $\neg(p \wedge q) \equiv \neg p \vee \neg q$ (c) $\neg(p \vee q) \equiv \neg p \vee \neg q$ (d) $\neg(\neg p) \equiv p$

12. which one of the following is correct for the truth value of $(p \wedge q) \rightarrow \neg p$

P	q	$(p \wedge q) \rightarrow \neg p$
T	T	(a)
T	F	(b)
F	T	(c)
F	F	(d)

	(1)	(2)	(3)	(4)
(a)	T	T	T	T
(b)	F	T	T	T
(c)	F	F	T	T
(d)	T	T	T	F

13. Which one of the following is a binary operation on N ?

(a) subtraction (b) Multiplication (c) division (d) all the above

14. In the set R of real numbers '$*$' is defined as follows. Which one of the following is not a binary operation on R?

(a) $a * b = \min(a, b)$ (b) $a * b = \max(a, b)$ (c) $a * b = a$ (d) $a * b = a^b$

15. If a compound statement involves 3 simple statements then the number of rows in the truth table is

(a) 9 (b) 8 (c) 6 (d) 3

16. Which one is the inverse of the statement $(p \vee q) \rightarrow (p \wedge q)$?

(a) $(p \wedge q) \rightarrow (p \vee q)$ (b) $\neg(p \vee q) \rightarrow (p \wedge q)$ (c) $(\neg p \vee \neg q) \rightarrow (\neg p \wedge \neg q)$ (d) $(\neg p \wedge \neg q) \rightarrow (\neg p \vee \neg q)$

17. Determine the truth value of each of the following statement?

(a) 4+2=5 and 6+3=9 (b) 3+2=5 and 6+1=7 (c) 4+5=9 and 1+2=4 (d) 3+2=5 and 4+7=11

	(a)	(b)	(c)	(d)
(1)	F	T	F	T
(2)	T	F	T	F
(3)	T	T	F	F
(4)	F	F	T	T

18. Which one of the following is not true?

(a) Negation of a negation of a statement is the statement itself

(b) If the last column of the truth table contains only T then it is a tautology

(c) If the last column of the truth table contains only F then it is a contradiction

(d) If p and q are any two statements then $p \leftrightarrow q$ is a tautology

19. The dual of $\neg(p \vee q) \vee [p \vee (p \wedge \neg r)]$ is

(a) $\neg(p \wedge q) \wedge [p \vee (p \wedge \neg r)]$ (b) $(p \wedge q) \wedge [p \wedge (p \vee \neg r)]$ (c) $\neg(p \wedge q) \wedge [p \wedge (p \wedge r)]$ (d) $\neg(p \wedge q) \wedge [p \wedge (p \vee \neg r)]$

20. Tthe proposition $p \wedge (\neg p \vee q)$ is

(a) a tautology (b) a contradiction (c) logically equivalent to $p \wedge q$ (d) logically equivalent to $p \vee q$

VECTOR ALGEBRA

Important hints:

- $\vec{a}\cdot\vec{b}=|\vec{a}||\vec{b}|\cos\theta$
- $\vec{a}\times\vec{b}=|\vec{a}||\vec{b}|\sin\theta\,\hat{n}$
- Work done $W=\vec{F}\cdot\vec{d}$
- Torque $\vec{\tau}=\vec{r}\times\vec{F}$
- $\vec{a},\vec{b}$ are perpendicular vectors $\Leftrightarrow \vec{a}\cdot\vec{b}=0$
- $\vec{a},\vec{b}$ are parallel vectors $\Leftrightarrow \vec{a}\times\vec{b}=\vec{0}$
- Volume of parallelepiped with coterminous vectors $V=\left|[\vec{a},\vec{b},\vec{c}]\right|$
- **If $\vec{a}=a_1\hat{\imath}+a_2\hat{\jmath}+a_3\hat{k}$ and $\overrightarrow{b}=b_1\hat{\imath}+b_2\hat{\jmath}+b_3\hat{k}$**

Scalar product (or) dot product

$\vec{a}\cdot\vec{b}=a_1b_1+a_2b_2+a_3b_3$

The vector product (or) cross product

$$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\end{vmatrix}$$

- If θ is the acute angle between two straight lines $\vec{r}=\vec{a}+s\vec{b}$ and $\vec{r}=\vec{c}+t\vec{d}$, then $\theta=\cos^{-1}\left(\frac{|\vec{b}\cdot\vec{d}|}{|\vec{b}||\vec{d}|}\right)$
- The acute angle θ between the two planes $\vec{r}\cdot\vec{n}_1=p_1$ and $\vec{r}\cdot\vec{n}_2=p_2$ is $\theta=\cos^{-1}\left(\frac{|\vec{n}_1\cdot\vec{n}_2|}{|\vec{n}_1||\vec{n}_2|}\right)$
- If θ is the acute angle between the line $\vec{r}=\vec{a}+t\vec{b}$ and the plane $\vec{r}\cdot\vec{n}=p$, then $\theta=\sin^{-1}\left(\frac{|\vec{b}\cdot\vec{n}|}{|\vec{b}||\vec{n}|}\right)$
- Let $\vec{a}=a_1\hat{\imath}+a_2\hat{\jmath}+a_3\hat{k}$, $\vec{b}=b_1\hat{\imath}+b_2\hat{\jmath}+b_3\hat{k}$,
- If two lines $\frac{x-x_1}{b_1}=\frac{y-y_1}{b_2}=\frac{z-z_1}{b_3}$ and $\frac{x-x_2}{d_1}=\frac{y-y_2}{d_2}=\frac{z-z_2}{d_3}$ intersect, then

$$\begin{vmatrix}x_2-x_1 & y_2-y_1 & z_2-z_1\\ b_1 & b_2 & b_3\\ d_1 & d_2 & d_3\end{vmatrix}=0$$

$\vec{c}=c_1\hat{\imath}+c_2\hat{\jmath}+c_3\hat{k}$. Then,

$\vec{a},\vec{b},\vec{c}$ are coplanar

$$\Leftrightarrow [\vec{a},\vec{b},\vec{c}]=0 \Leftrightarrow \begin{vmatrix}a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\\ c_1 & c_2 & c_3\end{vmatrix}=0$$

- MODEL-I

Parametric Vector Equation

$$\vec{r}=\vec{a}+s\vec{b}+t\vec{c}$$

Non Parametric Vector Equation

$$(\vec{r}-\vec{a})\cdot(\vec{b}\times\vec{c})=0$$

Cartesian Equation

$$\begin{vmatrix}x-x_1 & y-y_1 & z-z_1\\ b_1 & b_2 & b_3\\ c_1 & c_2 & c_3\end{vmatrix}=0$$

- MODEL-II

Parametric Vector Equation

$$\vec{r}=(1-s)\vec{a}+s\vec{b}+t\vec{c}$$

Non Parametric Vector Equation

$$(\vec{r}-\vec{a})\cdot[(\vec{b}-\vec{a})\times\vec{c}]=0$$

Cartesian Equation

$$\begin{vmatrix}x-x_1 & y-y_1 & z-z_1\\ x_2-x_1 & y_2-y_1 & z_2-z_1\\ c_1 & c_2 & c_3\end{vmatrix}=0$$

- MODEL III

Parametric Vector Equation

$$\vec{r}=(1-s-t)\vec{a}+s\vec{b}+t\vec{c}$$

Non Parametric Vector Equation

$$(\vec{r}-\vec{a})\cdot[(\vec{b}-\vec{a})\times(\vec{c}-\vec{a})]=0$$

Cartesian Equation

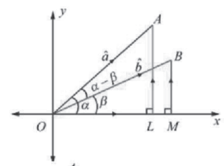
$$\begin{vmatrix}x-x_1 & y-y_1 & z-z_1\\ x_2-x_1 & y_2-y_1 & z_2-z_1\\ x_3-x_1 & y_3-y_1 & z_3-z_1\end{vmatrix}=0$$

MATHS

5 MARKS

1.By vector method, prove that
$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

__Soln:__

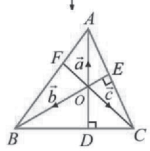


Let $\hat{a}$ and $\hat{b}$ are two unit vectors

$\hat{a} = \cos\alpha\hat{\imath} + \sin\alpha\hat{\jmath}$

$\hat{b} = \cos\beta\hat{\imath} + \sin\beta\hat{\jmath}$

$\hat{b}\cdot\hat{a} = \cos(\alpha-\beta) \dashrightarrow (1)$



$\hat{b}\cdot\hat{a} = (\cos\beta\hat{\imath} + \sin\beta\hat{\jmath})\cdot(\cos\alpha\hat{\imath} + \sin\alpha\hat{\jmath})$

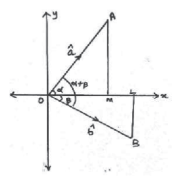
$= \cos\alpha\cos\beta + \sin\alpha\sin\beta \dashrightarrow (2)$

From (1)&(2)

$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

2.By vector method, prove that
$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

__Soln:__



Let $\hat{a}$ and $\hat{b}$ are two unit vectors

$\hat{a} = \cos\alpha\hat{\imath} + \sin\alpha\hat{\jmath}$

$\hat{b} = \cos\beta\hat{\imath} - \sin\beta\hat{\jmath}$

$\hat{b}\cdot\hat{a} = \cos(\alpha+\beta) \dashrightarrow (1)$

$\hat{b}\cdot\hat{a} = (\cos\beta\hat{\imath} - \sin\beta\hat{\jmath})\cdot(\cos\alpha\hat{\imath} + \sin\alpha\hat{\jmath})$

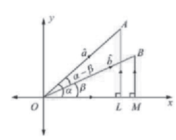
$= \cos\alpha\cos\beta - \sin\alpha\sin\beta \dashrightarrow (2)$

From (1)&(2)

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

3.By vector method, prove that
$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

__Soln:__



Let $\hat{a}$ and $\hat{b}$ are two unit vectors

$\hat{a} = \cos\alpha\hat{\imath} + \sin\alpha\hat{\jmath}$

$\hat{b} = \cos\beta\hat{\imath} + \sin\beta\hat{\jmath}$

$\hat{b}\times\hat{a} = \sin(\alpha-\beta)(\hat{k}) \dashrightarrow (1)$

$$\hat{b}\times\hat{a} = \begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ \cos\beta & \sin\beta & 0\\ \cos\alpha & \sin\alpha & 0\end{vmatrix}$$

$= (\sin\alpha\cos\beta - \cos\alpha\sin\beta)(\hat{k}) \dashrightarrow (2)$

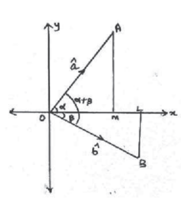
From (1) & (2)

$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

4.By vector method, prove that

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

__Soln:__



Let $\hat{a}$ and $\hat{b}$ are two unit vectors

$\hat{a} = \cos\alpha\hat{\imath} + \sin\alpha\hat{\jmath}$

$\hat{b} = \cos\beta\hat{\imath} - \sin\beta\hat{\jmath}$

$\hat{b}\times\hat{a} = \sin(\alpha+\beta)\hat{k} \dashrightarrow (1)$

$$\hat{b}\times\hat{a} = \begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ \cos\beta & -\sin\beta & 0\\ \cos\alpha & \sin\alpha & 0\end{vmatrix}$$

$= (\sin\alpha\cos\beta + \cos\alpha\sin\beta)\hat{k} \dashrightarrow (2)$

From (1)&(2)

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

5.Prove by vector method that the perpendiculars (attitudes) from the vertices to the opposite sides of a triangle are concurrent.

__Soln:__

$\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$, $\overrightarrow{OC} = \vec{c}$

AD⊥BC ; BE ⊥CA

To prove CF⊥BA

Case:1 AD⊥BC

$\overrightarrow{OA}\cdot\overrightarrow{BC} = 0$

$\overrightarrow{OA}\cdot(\overrightarrow{OC} - \overrightarrow{OB}) = 0$

$\vec{a}\cdot\vec{c} - \vec{a}\cdot\vec{b} = 0 \longrightarrow (1)$

Case:2 BE ⊥ CA

$\overrightarrow{OB}\cdot\overrightarrow{CA} = 0$

$\overrightarrow{OB}\cdot(\overrightarrow{OA} - \overrightarrow{OC}) = 0$

$\vec{b}\cdot\vec{a} - \vec{b}\cdot\vec{c} = 0 \longrightarrow (2)$

From (1) + (2)$\Rightarrow$ $\vec{a}\cdot\vec{c} - \vec{b}\cdot\vec{c} = 0$

$(\vec{a} - \vec{b})\cdot\vec{c} = 0$

$(\overrightarrow{OA} - \overrightarrow{OB})\cdot\overrightarrow{OC} = 0$

$\overrightarrow{BA}\cdot\overrightarrow{OC} = 0$

$\overrightarrow{BA}\cdot\overrightarrow{CF} = 0 \Rightarrow$ CF⊥ BA

Hence, the perpendiculars (attitudes) from the vertices to the opposite sides of a triangle are concurrent.

6.If $\vec{a} = \hat{\imath} - \hat{\jmath}, \vec{b} = \hat{\imath} - \hat{\jmath} - 4\hat{k}, \vec{c} = 3\hat{\jmath} - \hat{k}$,and $\vec{d} = 2\hat{\imath} + 5\hat{\jmath} + \hat{k}$ verify that
$(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d}) = [\vec{a},\vec{b},\vec{d}]\vec{c} - [\vec{a},\vec{b},\vec{c}]\vec{d}$

Soln:

$$\vec{a}\times\vec{b} = \begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ 1 & -1 & 0\\ 1 & -1 & -4\end{vmatrix} = 4\hat{\imath} + 4\hat{\jmath}$$

$$\vec{c}\times\vec{d} = \begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ 0 & 3 & -1\\ 2 & 5 & 1\end{vmatrix} = 8\hat{\imath} - 2\hat{\jmath} - 6\hat{k}$$

$$(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d}) = \begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ 4 & 4 & 0\\ 8 & -2 & -6\end{vmatrix}$$

$= -24\hat{\imath} + 24\hat{\jmath} - 40\hat{k} \dashrightarrow (1)$

$$[\vec{a},\vec{b},\vec{d}] = \begin{vmatrix}1 & -1 & 0\\ 1 & -1 & -4\\ 2 & 5 & 1\end{vmatrix} = 28\,,$$

$$[\vec{a},\vec{b},\vec{c}] = \begin{vmatrix}1 & -1 & 0\\ 1 & -1 & -4\\ 0 & 3 & -1\end{vmatrix} = 12$$

$[\vec{\mathbf{a}},\vec{\mathbf{b}},\vec{\mathbf{d}}]\vec{\mathbf{c}} - [\vec{\mathbf{a}},\vec{\mathbf{b}},\vec{\mathbf{c}}]\vec{\mathbf{d}} = \mathbf{28(3\hat{j} - \hat{k}) - 12(2\hat{i} + 5\hat{j} + \hat{k})}$

$= -24\hat{\imath} + 24\hat{\jmath} - 40\hat{k} \dashrightarrow (2)$

From (1), (2)

$$(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d}) = [\vec{a},\vec{b},\vec{d}]\vec{c} - [\vec{a},\vec{b},\vec{c}]\vec{d}$$

Try Yourself:

$$(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d}) = [\vec{a},\vec{c},\vec{d}]\vec{b} - [\vec{b},\vec{c},\vec{d}]\vec{a}$$

7.If $\vec{a} = 2\hat{\imath} + 3\hat{\jmath} - \hat{k}, \vec{b} = 3\hat{\imath} + 5\hat{\jmath} + 2\hat{k}$,and $\vec{c} = -\hat{\imath} - 2\hat{\jmath} + 3\hat{k}$ verify that $\vec{a}\times(\vec{b}\times\vec{c}) = (\vec{a}\cdot\vec{c})\vec{b} - (\vec{a}\cdot\vec{b})\vec{c}$

Soln:

$$\vec{b}\times\vec{c} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3 & 5 & 2 \\ -1 & -2 & 3 \end{vmatrix} = 19\hat{\imath} - 11\hat{\jmath} - \hat{k}$$

$$\vec{a}\times(\vec{b}\times\vec{c}) = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 3 & -1 \\ 19 & -11 & -1 \end{vmatrix}$$

$$= -14\hat{\imath} - 17\hat{\jmath} - 79 ---> (1)$$

$$\vec{a}\cdot\vec{c} = (2\hat{\imath} + 3\hat{\jmath} - \hat{k})\cdot(-\hat{\imath} - 2\hat{\jmath} + 3\hat{k}) = -11$$

$$\vec{a}\cdot\vec{b} = (2\hat{\imath} + 3\hat{\jmath} - \hat{k})\cdot(3\hat{\imath} + 5\hat{\jmath} + 2\hat{k}) = 19$$

$(\vec{a}\cdot\vec{c})\vec{b} - (\vec{a}\cdot\vec{b})\vec{c}$

$$= -11(3\hat{\imath} + 5\hat{\jmath} + 2\hat{k}) - 19(-\hat{\imath} - 2\hat{\jmath} + 3\hat{k})$$

$(\vec{a}\cdot\vec{c})\vec{b} - (\vec{a}\cdot\vec{b})\vec{c} = -14\hat{\imath} - 17\hat{\jmath} - 79\hat{k} \quad ---> (2)$

From (1),(2)

$$\vec{a}\times(\vec{b}\times\vec{c}) = (\vec{a}\cdot\vec{c})\vec{b} - (\vec{a}\cdot\vec{b})\vec{c}$$

Try yourself :

$$(\vec{a}\times\vec{b})\times\vec{c} = (\vec{a}\cdot\vec{c})\vec{b} - (\vec{b}\cdot\vec{c})\vec{a}$$

MODEL- I

8.Find the non-parametric form of Vector Equation, and Cartesian equation of the plane passing through the point (0,1,-5) and parallel to the straight lines $\vec{r} = (\hat{\imath} + 2\hat{\jmath} - 4\hat{k}) + s(2\hat{\imath} + 3\hat{\jmath} + 6\hat{k})$ and $\vec{r} = (\hat{\imath} - 3\hat{\jmath} + 5\hat{k}) + t(\hat{\imath} + \hat{\jmath} - \hat{k})$.

Soln:

$\vec{a} = 0\hat{\imath} + \hat{\jmath} - 5\hat{k} \quad \vec{b} = 2\hat{\imath} + 3\hat{\jmath} + 6\hat{k} \quad \vec{c} = \hat{\imath} + \hat{\jmath} - \hat{k}$

Vector Equation: $\vec{r} = \vec{a} + s\vec{b} + t\vec{c}$

$$\vec{r} = (0\hat{\imath} + \hat{\jmath} - 5\hat{k}) + s(2\hat{\imath} + 3\hat{\jmath} + 6\hat{k}) + t(\hat{\imath} + \hat{\jmath} - \hat{k})$$

Cartesian Equation: $\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = 0$

$$\begin{vmatrix} x - 0 & y - 1 & z + 5 \\ 2 & 3 & 6 \\ 1 & 1 & -1 \end{vmatrix} = 0$$

$$(x - 0)(-3 - 6) - (y - 1)(-2 - 6) + (z + 5)(2 - 3) = 0$$

$$-9x + 8y - z - 13 = 0$$

or $\quad 9x - 8y + z + 13 = 0$

Non Parametirc Vector Equation:

$$(\vec{r} - \vec{a})\cdot(\vec{b}\times\vec{c}) = 0$$

$$\vec{r}\cdot(9\hat{\imath} - 8\hat{\jmath} + \hat{k}) + 13 = 0$$

9.Find the non-parametric form of Vector Equation, and Cartesian equation of the plane passing through the point (2,3,6) and parallel to the straight lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-3}{1}$ and $\frac{x+3}{2} = \frac{y-3}{-5} = \frac{z+1}{-3}$

Soln:

$\vec{a} = 2\hat{\imath} + 3\hat{\jmath} + 6\hat{k} \quad \vec{b} = 2\hat{\imath} + 3\hat{\jmath} + \hat{k} \quad \vec{c} = 2\hat{\imath} - 5\hat{\jmath} - 3\hat{k}$

Vector Equation: $\vec{r} = \vec{a} + s\vec{b} + t\vec{c}$

$$\vec{r} = (2\hat{\imath} + 3\hat{\jmath} + 6\hat{k}) + s(2\hat{\imath} + 3\hat{\jmath} + \hat{k}) + t(2\hat{\imath} - 5\hat{\jmath} - 3\hat{k})$$

Cartesian Equation: $\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = 0$

$$\begin{vmatrix} x - 2 & y - 3 & z - 6 \\ 2 & 3 & 1 \\ 2 & -5 & -3 \end{vmatrix} = 0$$

$$(x - 2)(-9 + 5) - (y - 3)(-6 - 2) + (z - 6)(-10 - 6) = 0$$

$$-4x + 8y - 16z + 80 = 0$$

(or) $\quad x - 2y + 4z - 20 = 0$

Non Parametirc.Vector Equation:

$$(\vec{r} - \vec{a})\cdot(\vec{b}\times\vec{c}) = 0$$

$$\Rightarrow \vec{r}\cdot(\hat{\imath} - 2\hat{\jmath} + 4\hat{k}) - 20 = 0$$

10. Find the non-parametric form of Vector Equation and Cartesian equation of the plane passing through the point (1,-2,4) and perpendicular to the plane $x + 2y - 3z = 11$ and parallel to the line $\frac{x+7}{3} = \frac{y+3}{-1} = \frac{z}{1}$.

Soln:

$\vec{a} = \hat{\imath} - 2\hat{\jmath} + 4\hat{k} \quad \vec{b} = \hat{\imath} + 2\hat{\jmath} - 3\hat{k} \quad \vec{c} = 3\hat{\imath} - \hat{\jmath} + \hat{k}$

Vector Equation : $\vec{r} = \vec{a} + s\vec{b} + t\vec{c}$

$$\vec{r} = (\hat{\imath} - 2\hat{\jmath} + 4\hat{k}) + s(\hat{\imath} + 2\hat{\jmath} - 3\hat{k}) + t(3\hat{\imath} - \hat{\jmath} + \hat{k})$$

Cartesian Equation: $\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = 0$

$$\Rightarrow \begin{vmatrix} x - 1 & y + 2 & z - 4 \\ 1 & 2 & -3 \\ 3 & -1 & 1 \end{vmatrix} = 0$$

$$(x - 1)(2 - 3) - (y + 2)(1 + 9) + (z - 4)(-1 - 6) = 0$$

$$-x - 10y - 7z + 9 = 0 \quad (or)$$

$$x + 10y + 7z - 9 = 0$$

Non Parametirc.Vector Equation:

$$(\vec{r} - \vec{a})\cdot(\vec{b}\times\vec{c}) = 0$$

$$\vec{r}\cdot(\hat{\imath} + 10\hat{\jmath} + 7\hat{k}) - 9 = 0$$

MATHS

11.Find the parametric form of Vector Equation, & Cartesian equations of the plane containing the line $\vec{r} = (\hat{\imath} - \hat{\jmath} + 3\hat{k}) + t(2\hat{\imath} - \hat{\jmath} + 4\hat{k})$ and perpendicular to plane $\vec{r}.(\hat{\imath} + 2\hat{\jmath} + \hat{k}) = 8$.

Soln: $\vec{a} = \hat{\imath} - \hat{\jmath} + 3\hat{k}$ $\vec{b} = 2\hat{\imath} - \hat{\jmath} + 4\hat{k}$ $\vec{c} = \hat{\imath} + 2\hat{\jmath} + \hat{k}$

Vector Equation : $\vec{r} = \vec{a} + s\vec{b} + t\vec{c}$

$$\vec{r} = (\hat{\imath} - \hat{\jmath} + 3\hat{k}) + s(2\hat{\imath} - \hat{\jmath} + 3\hat{k}) + t(\hat{\imath} + 2\hat{\jmath} + \hat{k})$$

Cartesian Equation: $\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = 0$

$$\begin{vmatrix} x - 1 & y + 1 & z - 3 \\ 2 & -1 & 4 \\ 1 & 2 & 1 \end{vmatrix} = 0$$

$$(x - 1)(-1 - 8) - (y + 1)(2 - 4) + (z - 3)(4 + 1) = 0$$

$$-9x + 2y + 5z - 4 = 0$$

$$(or)\ \ 9x - 2y - 5z + 4 = 0$$

Non Parametirc.Vector Equation: $(\vec{r} - \vec{a}) \cdot (\vec{b} \times \vec{c}) = 0$

$$\Rightarrow \vec{r} \cdot (9\hat{\imath} - 2\hat{\jmath} - 5\hat{k}) + 4 = 0$$

12.Find the non-parametric form of vector eqn, and Cartesian eqns of the plane $\vec{r} = (6\hat{\imath} - \hat{\jmath} + \hat{k}) + s(-\hat{\imath} + 2\hat{\jmath} + \hat{k}) + t(-5\hat{\imath} - 4\hat{\jmath} - 5\hat{k})$.

Soln:

$\vec{a} = 6\hat{\imath} - \hat{\jmath} + \hat{k}$ $\vec{b} = -\hat{\imath} + 2\hat{\jmath} + \hat{k}$ $\vec{c} = -5\hat{\imath} - 4\hat{\jmath} - 5\hat{k}$

Vector Equation : $\vec{r} = \vec{a} + s\vec{b} + t\vec{c}$

$$\vec{r} = (6\hat{\imath} - \hat{\jmath} + \hat{k}) + s(-\hat{\imath} + 2\hat{\jmath} + \hat{k}) + t(-5\hat{\imath} - 4\hat{\jmath} - 5\hat{k})$$

Cartesian Equation: $\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = 0$

$$\begin{vmatrix} x - 6 & y + 1 & z - 1 \\ -1 & 2 & 1 \\ -5 & -4 & -5 \end{vmatrix} = 0$$

$$(x - 6)(-10 + 4) - (y + 1)(5 + 5) + (z - 1)(4 + 10) = 0$$

$$-6x - 10y + 14z + 12 = 0\ \ (or)$$

$$3x + 5y - 7z - 6 = 0$$

Non Parametirc.Vector Equation:

$$(\vec{r} - \vec{a}) \cdot (\vec{b} \times \vec{c}) = 0$$

$$\Rightarrow\ \vec{r} \cdot (3\hat{\imath} + 5\hat{\jmath} - 7\hat{k}) - 6 = 0$$

MODEL-II

13.Find the non-parametric and Cartesian form of the eqn of the plane passing through the points (-1,2,0), (2,2,-1) and parallel to the straight line $\frac{x-1}{1} = \frac{2y+1}{2} = \frac{z+1}{-1}$

Soln:

$\vec{a} = -\hat{\imath} + 2\hat{\jmath} + 0\hat{k}$ $\vec{b} = 2\hat{\imath} + 2\hat{\jmath} - \hat{k}$ $\vec{c} = \hat{\imath} + \hat{\jmath} - \hat{k}$

Vector Equation: $\vec{r} = (1 - s)\vec{a} + s\vec{b} + t\vec{c}$

$$\vec{r} = (1 - s)(-\hat{\imath} + 2\hat{\jmath}) + s(2\hat{\imath} + 2\hat{\jmath} - \hat{k}) + t(\hat{\imath} + \hat{\jmath} - \hat{k})$$

Cartesian Equation: $\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ c_1 & c_2 & c_3 \end{vmatrix} = 0$

$$\begin{vmatrix} x + 1 & y - 2 & z - 0 \\ 3 & 0 & -1 \\ 1 & 1 & -1 \end{vmatrix} = 0$$

$$(x + 1)(0 + 1) - (y - 2)(-3 + 1) + (z - 0)(3 - 0) = 0$$

$$x + 2y + 3z - 3 = 0$$

Non Parametirc Vector Equation:

$$(\vec{r} - \vec{a}) \cdot [(\vec{b} - \vec{a}) \times \vec{c}] = 0$$

$$\vec{r} \cdot (\hat{\imath} + 2\hat{\jmath} + 3\hat{k}) = 3$$

14.Find the non-parametric form of vector eqn, Cartesian eqns of the plane passing through the points (2,2,1), (9,3,6) and perpendicular to the plane $2x + 6y + 6z = 9$.

Soln:

$\vec{a} = 2\hat{\imath} + 2\hat{\jmath} + \hat{k}$ $\vec{b} = 9\hat{\imath} + 3\hat{\jmath} + 6\hat{k}$ $\vec{c} = 2\hat{\imath} + 6\hat{\jmath} + 6\hat{k}$

Vector Equation: $\vec{r} = (1 - s)\vec{a} + s\vec{b} + t\vec{c}$

$$\vec{r} = (1 - s)(2\hat{\imath} + 2\hat{\jmath} + \hat{k}) + s(9\hat{\imath} + 3\hat{\jmath} + 6\hat{k}) + t(2\hat{\imath} + 6\hat{\jmath} + 6\hat{k})$$

Cartesian Equation: $\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ c_1 & c_2 & c_3 \end{vmatrix} = 0$

$$\begin{vmatrix} x - 2 & y - 2 & z - 1 \\ 7 & 1 & 5 \\ 2 & 6 & 6 \end{vmatrix} = 0$$

$$(x - 2)(6 - 30) - (y - 2)(42 - 10) + (z - 1)(42 - 2) = 0$$

$$-24x - 32y + 40z + 72 = 0$$

$$(or)\ \ 3x + 4y - 5z - 9 = 0$$

Non Parametirc Vector Equation:

$$(\vec{r} - \vec{a}) \cdot [(\vec{b} - \vec{a}) \times \vec{c}] = 0$$

$$\vec{r} \cdot (3\hat{\imath} + 4\hat{\jmath} - 5\hat{k}) - 9 = 0$$

15.Find parametric form of Vector Equation and Cartesian equations of the plane passing through the points (2, 2,1), (1,-2,3) and parallel to the straight line passing through the points (2, 1, −3) and (−1,5, −8).

Soln:

$\vec{a} = 2\hat{\imath} + 2\hat{\jmath} + \hat{k}$ $\vec{b} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k}$ $\vec{c} = -3\hat{\imath} + 4\hat{\jmath} - 5\hat{k}$

Vector Equation: $\vec{r} = (1 - s)\vec{a} + s\vec{b} + t\vec{c}$

$$\vec{r} = (1 - s)(2\hat{\imath} + 2\hat{\jmath} + \hat{k}) + s(\hat{\imath} - 2\hat{\jmath} + 3\hat{k}) + t(-3\hat{\imath} + 4\hat{\jmath} - 5\hat{k})$$

Cartesian Equation: $\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ c_1 & c_2 & c_3 \end{vmatrix} = 0$

$$\begin{vmatrix} x - 2 & y - 2 & z - 1 \\ -1 & -4 & 2 \\ -3 & 4 & -5 \end{vmatrix} = 0$$

$$(x - 2)(20 - 8) - (y - 2)(5 + 6) + (z - 1)(-4 - 12) = 0$$

$$12x - 11y - 16z + 14 = 0$$

Non Parametirc Vector Equation:

$$(\vec{r} - \vec{a}) \cdot [(\vec{b} - \vec{a}) \times \vec{c}] = 0$$

$$\vec{r} \cdot (12\hat{\imath} - 11\hat{\jmath} - 16\hat{k}) + 14 = 0$$

MODEL-III

16.Find the parametric vector, non-parametric vector and Cartesian form of the equations of the plane passing through the three non-Collinear points $(3,6,-2)$, $(-1,-2,6)$,and $(6,4,-2)$.

<u>Soln</u>:

$\vec{a} = 3\hat{\imath} + 6\hat{\jmath} - 2\hat{k} \quad \vec{b} = -\hat{\imath} - 2\hat{\jmath} + 6\hat{k} \quad \vec{c} = 6\hat{\imath} + 4\hat{\jmath} - 2\hat{k}$

Vector Equation: $\vec{r} = (1 - s - t)\vec{a} + s\vec{b} + t\vec{c}$

$\vec{r} = (1 - s - t)(3\hat{i} + 6\hat{j} - 2\hat{k}) + s(-\hat{i} - 2\hat{j} + 6\hat{k}) + t(6\hat{i} - 4\hat{j} - 2\hat{k})$

Cartesian Equation: $\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$

$$\begin{vmatrix} x - 3 & y - 6 & z + 2 \\ -4 & -8 & 8 \\ 3 & -2 & 0 \end{vmatrix} = 0$$

$$(x - 3)(0 + 16) - (y - 6)(0 - 24) + (z + 2)(8 + 24) = 0$$

$$16x - 48 + 24y - 144 + 32z + 64 = 0$$

$$(or) \quad 16x + 24y + 32z - 128 = 0$$

$$2x + 3y + 4z - 16 = 0$$

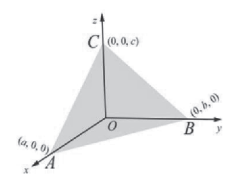
Non Parametirc Vector Equation:

$$(\vec{r} - \vec{a}) \cdot [(\vec{b} - \vec{a}) \times (\vec{c} - \vec{a})] = 0$$

$$\vec{r} \cdot (2\hat{i} + 3\hat{j} + 8\hat{k}) - 16 = 0$$

17. Derive the equation of the plane in the intercept form.

Soln:



$A(a,0,0), B(0,b,0), C(0,0,c)$

$\vec{a} = a\hat{\imath} + 0\hat{\jmath} + 0\hat{k}$,

$\vec{b} = 0\hat{\imath} + b\hat{\jmath} + 0\hat{k}$

and $\vec{c} = 0\hat{\imath} + 0\hat{\jmath} + c\hat{k}$,

Vector Equation: $\vec{r} = (1 - s - t)\vec{a} + s\vec{b} + t\vec{c}$

$$\vec{r} = (1 - s - t)a\hat{\imath} + sb\hat{\jmath} + tc\hat{k}$$

Cartesian Equation: $\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$

$$\begin{vmatrix} x - a & y & z \\ -a & b & 0 \\ -a & 0 & c \end{vmatrix} = 0$$

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

18. **Show that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$ intersect and hence find the point of intersection.**

Soln:

Condition for intersecting lines

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$$

$$\begin{vmatrix} 3 & -1 & -3 \\ 2 & 3 & 4 \\ 5 & 2 & 1 \end{vmatrix} = 0$$

Let $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = s$

$\Longrightarrow \quad (x, y, z) = (2s + 1, 3s + 2, 4s + 3)$

$\frac{x-4}{5} = \frac{y-1}{2} = z = t$

$\Longrightarrow \quad (x, y, z) = (5t + 4, 2t + 1, t)$

At the point of intersection

$$(2s + 1, 3s + 2, 4s + 3) = (5t + 4, 2t + 1, t)$$

$$\therefore \text{we get } s = -1, \quad t = -1$$

The point of intersection $(x, y, z) = (-1, -1, -1)$

Try yourself.

Show that the lines $\frac{x-3}{3} = \frac{y-3}{-1}, z - 1 = 0$ and $\frac{x-6}{2} = \frac{z-1}{3}, y - 2 = 0$ intersect and hence find the point of intersection. **Hint:** $\frac{x-3}{3} = \frac{y-3}{-1} = \frac{z-1}{0}$ **&** $\frac{x-6}{2} = \frac{z-1}{3} = \frac{y-2}{0}$

Find the parametric form of a vector equation of a straight line passing through the point of intersection of the straight lines $\vec{r} = \hat{\imath} + 3\hat{\jmath} - \hat{k} + t(2\hat{\imath} + 3\hat{\jmath} + 2\hat{k})$ and $\frac{x-2}{1} = \frac{y-4}{2} = \frac{z+3}{4}$, and perpendicular to both straight lines. **Hint:** $\frac{x-1}{2} = \frac{y-3}{3} = \frac{z+1}{2}$ **&** $\frac{x-2}{1} = \frac{y-4}{2} = \frac{z+3}{4}$

If the two lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-m}{2} = z$ intersect at a point, find the value of m.

ANALYTICAL GEOMETRY

5 Marks

Hints:

Equation of circle with centre(0,0)and radius r

$$x^2 + y^2 = r^2$$

Equation of circle with end ponts (x_1, y_1)&(x_2, y_2)

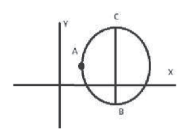
$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

Elipse $c^2 = a^2m^2 + b^2$, point of contact $\left(-\frac{a^2m}{c}, \frac{b^2}{c}\right)$

Hyperbola $c^2 = a^2m^2 - b^2$, point of contact $\left(-\frac{a^2m}{c}, -\frac{b^2}{c}\right)$

1.Find the equation of the circle passing through the points $(1, 1)$, $(2, -1)$, and $(3, 2)$

Soln:



$A(1,1), B(2,-1), C(3,2)$

$M_1 = \text{Slope of } AB = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-1}{2-1} = -2$

$M_2 = \text{Slope of } AC = \frac{2-1}{3-1} = \frac{1}{2}$

$m_1 \times m_2 = -1 \;\therefore \angle A = 90°$

End points of diameter B, C,the eqn of circle

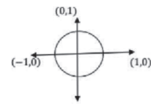
$$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$$

$$(x-2)(x-3)+(y+1)(y-2)=0$$

$$\boldsymbol{x^2+y^2-5x-y+4=0}$$

2.Find the equation of the circle through the points (1,0),(-1,0), and (0,1).

Soln :



End point of diameter of $(1,0),(-1,0)$

Centre(0,0), radius=1

Equation of circle $\boldsymbol{x^2+y^2=1}$

3.Show that the line $x-y+4=0$ is a tangent to the ellipse $x^2+3y^2=12$.Also find the coordinates of the point of contact.

Soln :

$x-y+4=0 \qquad x^2+3y^2=12$

$y=x+4 \qquad \frac{x^2}{12}+\frac{y^2}{4}=1$

$\boldsymbol{m=1, c=4} \qquad \boldsymbol{a^2=12, b^2=4}$

Condition:$\boldsymbol{c^2=a^2m^2+b^2}$

$$c^2=16=a^2m^2+b^2$$

$x-y+4=0$ is a tangent to $x^2+3y^2=12$

Point of contact: $\left(-\frac{a^2m}{c},\frac{b^2}{c}\right)=\boldsymbol{(-3,1)}$

4.Show that the line $5x+12y=9$ is a tangent to the hyperbola $x^2-9y^2=9$,also find point of contact?

Soln :

$5x+12y=9 \qquad x^2-9y^2=9$

$\Rightarrow y=-\frac{5}{12}x+\frac{3}{4}, \qquad \Rightarrow \frac{x^2}{9}-\frac{y^2}{1}=9$

$\boldsymbol{m=-\frac{5}{12}, c=\frac{3}{4}} \qquad \boldsymbol{a^2=9, b^2=1}$

Condition $\boldsymbol{c^2=a^2m^2-b^2}$,

$$\Rightarrow \boldsymbol{c^2=\frac{9}{16}=a^2m^2-b^2}$$

$5x+12y=9$ is a tangent to the $x^2-9y^2=9$

Point of contact is $\left(-\frac{a^2m}{c},-\frac{b^2}{c}\right)=\boldsymbol{(5,-\frac{4}{3})}$

5.A bridge has a parabolic arch that is 10m high in the centre and 30m wide at the bottom. Find the height of the arch 6m from the centre, on either sides.

Soln: $x^2=-4ay----\rightarrow(1)$

At $(15,-10)$

$(1)\Rightarrow(15)^2=-4a(-10)$

$\Rightarrow a=\frac{225}{40}$

$(1)\Rightarrow x^2=-4\left(\frac{225}{40}\right)y----\rightarrow(2)$

At $(6,-y_1)$

$(1)\Rightarrow(6)^2=-4\times\frac{225}{40}(-y_1)$

$\frac{36\times40}{4\times225}=y_1\Rightarrow y_1=1.6$

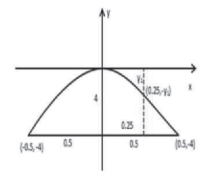
Required height is $\boldsymbol{10-y_1=10-1.6=8.4}$ **m**

6.At a water fountain, water attains a maximum height of 4m at horizontal distance of 0.5 m from its origin. If the path of water is a parabola, find the height of water at a horizontal distance of 0.75 m from the point of origin.

Soln:

$x^2=-4ay---\rightarrow(1)$

At $(-0.5,-4)$



$(1)\Rightarrow(-\frac{1}{2})^2=-4a(-4)\Rightarrow a=\frac{1}{64}$

$(1)\Rightarrow x^2=-4\left(\frac{1}{64}\right)y--\rightarrow(2)$

At $(0.25,-y_1)$

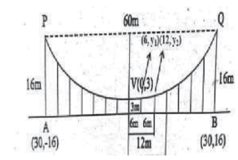
$(2)\Rightarrow\left(\frac{1}{4}\right)^2=-4\times\frac{1}{64}(-y_1)\Rightarrow\frac{64}{4\times16}=y_1\Rightarrow y_1=1$

Required distance is $\boldsymbol{4-y_1=4-1=3}$ **m**

7.Parabolic cable of a 60m portion of the roadbed of a suspension bridge are positioned as shown below. Vertical Cables are to be spaced every 6m along this portion of the roadbed. Calculate the lengths of first two of these vertical cables from the vertex.

Soln:

$x^2=4ay--------\rightarrow(1)$



At $(30,13)\Rightarrow30^2=4a(13)$

$\Rightarrow a=\frac{900}{52}$

$(1)\Rightarrow x^2=4\times\frac{900}{52}y$

$\Rightarrow x^2=\frac{900}{13}y----\rightarrow(2)$

(i)At $(6,y_1)$

$(2)\Rightarrow6^2=\frac{900}{13}y_1\Rightarrow\frac{36\times13}{900}=y_1\Rightarrow y_1=0.52$

Height of the first cable is $3+y_1=3+0.52=3.52$

(ii)At $(12,y_2)$

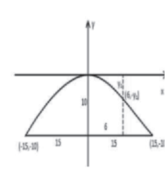
$(2)\Rightarrow12^2=\frac{900}{13}y_2\Rightarrow\frac{144\times13}{900}=y_2\Rightarrow\boldsymbol{y_2=2.08}$

Height of the second cable is

$$\boldsymbol{3+y_2=3+2.08=5.08}\text{ m}$$

8.Assume that water issuing from the end of a horizontal pipe, 7.5 m above the ground, describes a parabolic path. The vertex of the parabolic path is at the end of the pipe. At a position 2.5 m below the line of the pipe, the flow of water has curved outward 3m beyond the vertical line through the end of the pipe. How far beyond this vertical line will the water strike the ground?

Soln:



$x^2=-4ay---\rightarrow(1)$

At $(3, -2.5)$,

$(1) \Rightarrow (3)^2 = -4a(-2.5)$

$\Rightarrow a = \frac{9}{10}$

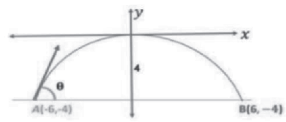
$(1) \Rightarrow \quad x^2 = -4\left(\frac{9}{10}\right) y - - \longrightarrow (2)$

At $(x_1, -7.5)$ $(2) \Rightarrow (x_1)^2 = -4 \times \frac{9}{10}(-7.5)$

$$\Rightarrow (\boldsymbol{x_1})^2 = \mathbf{9 \times 3} \Rightarrow \boldsymbol{x_1} = \mathbf{3\sqrt{3}\ m}$$

9.On lighting a rocket cracker it gets projected in a parabolic path and reaches a maximum height of 4*m* when it is 6*m* away from the point of projection. Finally it reaches the ground 12*m* away from the starting point. Find the angle of projection

Soln:



$x^2 = -4ay - - - \longrightarrow (1)$

At $(6, -4)$

$(1) \Rightarrow (6)^2 = -4a(-4) \Rightarrow a = \frac{36}{16} = \frac{9}{4}$

$(1) \Rightarrow x^2 = -4ay \Rightarrow x^2 = -4\left(\frac{9}{4}\right) y$

$\Rightarrow x^2 = -9y - - - \longrightarrow (2)$

$(2)\ diff.w.r.t.'x' \Rightarrow 2x = -9\frac{dy}{dx}$

$\therefore \frac{dy}{dx} = \frac{2x}{-9}$

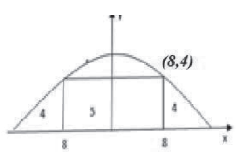
At $(-6, -4) \Rightarrow \frac{dy}{dx} = \frac{2(-6)}{-9}$

$\frac{dy}{dx} = tan\theta = \frac{4}{3}$

$\therefore \boldsymbol{\theta = tan^{-1}\left(\frac{4}{3}\right)}$

10.A tunnel through a mountain for a four lane highway is to have a elliptical opening. The total width of the highway (not the opening) is to be 16*m*, and the height at the edge of the road must be sufficient for a truck 4*m* high to clear if the highest point of the opening is to be 5*m* approximately. How wide must the opening be?

Soln:



$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 - - - \longrightarrow (1)$

Given $b = 5$

$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{5^2} = 1 - - \longrightarrow (2)$

At $(8,4)$ $\quad (2) \Rightarrow \frac{8^2}{a^2} + \frac{4^2}{5^2} = 1$

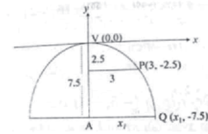
$\frac{8^2}{a^2} = 1 - \frac{16}{25} = \frac{25-1}{25} = \frac{9}{25} = \left(\frac{3}{5}\right)^2$

$\Rightarrow \frac{8}{a} = \frac{3}{5} \Rightarrow a = \frac{40}{3}$

Required opening is $2a = \frac{80}{3} = 26.66$ m

11.The maximum and minimum distances of the Earth from the Sun respectively are 152×10^6 km and 94.5×10^6 km. The Sun is at one focus of the elliptical orbit. Find the distance from the Sun to the other focus.

Soln.



$SA' = a + c = 152 \times 10^6$

$SA = a - c = 94.5 \times 10^6$

Subtracting $2c = 57.5 \times 10^6 =$

575×10^5 km

Distance from the Sun to the other focus is

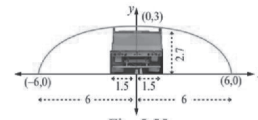
$$SS' = 575 \times 10^5 \text{ km}.$$

12.A semielliptical archway over a one-way road has a height of 3 m and a width of 12 m. The truck has a width of 3 m and a height of 2.7 m. Will the truck clear the opening of the archway?

Soln.

From the diagram $a = 6$ and $b = 3$

Equation of ellipse as $\frac{x^2}{6^2} + \frac{y^2}{3^2} = 1 - - - \longrightarrow (1)$



$At(\frac{3}{2}, y_1) \quad (1) \Rightarrow \frac{\left(\frac{3}{2}\right)^2}{36} + \frac{y^2}{9} = 1$

$y_1^2 = 9\left(1 - \frac{9}{144}\right)$

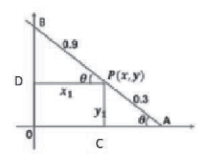
$y_1^2 = \frac{135}{16}$

$y_1 = \frac{\sqrt{135}}{4} = 2.90m$

The truck will clear the archway.

13.A rod of length 1.2 m moves with its ends always touching the coordinate axes. The locus of the point P on the rod 0.3 m from the end in contact with x-axis is an ellipse. Find the eccentricity.

Soln:



Right angle triangle PAC

$sin\theta = \frac{y_1}{0.3} \Rightarrow sin^2\theta = \frac{{y_1}^2}{0.09} - - \longrightarrow (1)$

Right angle triangle BPD

$cos\theta = \frac{x}{0.9} \Rightarrow cos^2\theta = \frac{{x_1}^2}{0.81} - - \longrightarrow (2)$

$(1)^2 + (2)^2 \Rightarrow \frac{{x_1}^2}{0.81} + \frac{{y_1}^2}{0.09} = cos^2\theta + sin^2\theta = 1$

The locus of $(x_1, y_1) is \quad \frac{x^2}{0.81} + \frac{y^2}{0.09} = 1.$

This is ellipse

Eccentricity $e = \sqrt{\frac{a^2 - b^2}{a^2}}$

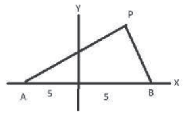
$= \sqrt{\frac{0.81 - 0.09}{0.81}} = \sqrt{\frac{0.72}{0.81}}$

$= \sqrt{\frac{8}{9}} = \frac{2\sqrt{2}}{3}$ m

14.Points *A* and *B* are 10*km* apart and it is determined from the sound of an explosion heard at those points at different times that the location of the explosion is 6 *km* closer to *A* than *B*. Show that the location of the explosion is restricted to a particular curve and find an equation of it.

MATHS

Soln:



$2ae = 10 \Rightarrow ae = 5$;

$2a = 6 \Rightarrow a = 3$

$3e = 5 \Rightarrow e = \frac{5}{3} > 1,$

$\therefore$ The curve is an hyperbola.

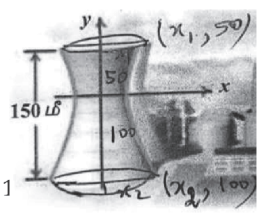
$b^2 = a^2(e^2-1) \Rightarrow b^2 = 9\left(\frac{25}{9}-1\right)$

$\Rightarrow b^2 = 9\left(\frac{25-9}{9}\right) \Rightarrow b^2 = 16$

Equation of hyperbola is $\frac{x^2}{a^2}-\frac{y^2}{b^2} = 1 \Rightarrow \frac{x^2}{9}-\frac{y^2}{16} = 1$

15.Cross section of a Nuclear cooling tower is in the shape of a hyperbola with equation $\frac{x^2}{30^2}-\frac{y^2}{44^2} = 1$ The tower is $150m$ tall and the distance from the top of the tower to the centre of the hyperbola is half the distance from the base of the tower to the centre of the hyperbola. Find the diameter of the top and base of the tower.

Soln:



Given $\frac{x^2}{30^2}-\frac{y^2}{44^2} = 1 \longrightarrow (1)$

At $(x_1, 50)$

$(1) \Rightarrow \frac{(x_1)^2}{30^2}-\frac{(50)^2}{44^2} = 1 \Rightarrow \frac{(x_1)^2}{30^2} = 1$

$x_1 = 45.41m$

$\therefore$ **the diameter of the top is $2x_1 = 90.82$m**

At $(x_2, 100)$

$(1) \Rightarrow \frac{(x_2)^2}{30^2}-\frac{(100)^2}{44^2} = 1 \Rightarrow \frac{(x_2)^2}{30^2} = 1+\frac{(100)^2}{44^2}$

$x_2 = 74.49m$

$\therefore$ **the diameter of the base is $2x_2 = 148.98m$**

COMPLEX NUMBERS

Important Hints:

1. $i = \sqrt{-1}, i^2 = -1,\ i^3 = -i,\ i^4 = 1,\ i^{4n} = 1$
2. Rectangular form of a complex number is x + iy real part is x, Imaginary part is y.
3. The conjugate of the complex number z = x + iy is $\bar{z}$= x −iy
4. If $z = x +$ iy then modulus of z is $|z| = \sqrt{x^2+y^2}$
5. Triangle inequality: For any two complex number z_1 and z_2 , $|z_1+z_2| \le |z_1|+|z_2|$& $||z_1|-|z_2|| \le |z_1+z_2| \le |z_1|+|z_2|$
6. $\sqrt{a \pm ib} = \pm\left[\sqrt{\frac{|z|+a}{2}} \pm i\sqrt{\frac{|z|-a}{2}}\right]$
7. Additive inverse of **$z = -z$** ,
   Multiplicative inverse of **z is $\frac{1}{z} = \bar{z}$**
8. z is real if any only if z = $\bar{z}$ and
   z is purely imaginary if and only if z = $-\bar{z}$
9. Distance between two complex numbers, z_1 and z_2 is $|z_1 - z_2|$
10. $|z - z_0| = r$ is the complex form of the equation of a circle. Centre is z_0 and radius is r.

5 Marks

1.If $z = x + iy$ is a complex number such that $\left|\frac{z-4i}{z+4i}\right| = 1$, S.T. the locus of z is real axis.

Soln:

$z = x + iy$

$\left|\frac{z-4i}{z+4i}\right| = 1 \Rightarrow \quad |z-4i| = |z+4i|$

$|x+iy-4i| = |x+iy+4i|$

$|x+i(y-4)|^2 = |x+i(y+4)|^2$

$x^2+(y-4)^2 = x^2+(y+4)^2$

$y = 0$

$\therefore$ ***z is real.***

2. If $z = x + iy$ is a complex number such that $Im\left(\frac{2z+1}{iz+1}\right) = 0$, show that the locus of z is $2x^2+2y^2+x-2y = 0$.

Soln:

Given $Im\left(\frac{2z+1}{iz+1}\right) = 0$, put $z = x+iy$

$Im\left(\frac{2(x+iy)+1}{i(x+iy)+1}\right) = 0$

$Im\left(\frac{2x+i2y+1}{ix+i^2y+1}\right) = 0 \qquad Im\left(\frac{a+ib}{c+id}\right) = \frac{bc-ad}{c^2+d^2}$

$Im\left(\frac{(2x+1)+i2y}{(1-y)+ix}\right) = 0$

$\left(\frac{2y(1-y)-x(2x+1)}{(1-y)^2+x^2}\right) = 0$

$2y - 2x^2 - 2y^2 - x = 0$ (or)

$2x^2 + 2y^2 + x - 2y = 0$

3. If $z = x + iy$ is a complex number such that $Re\left(\frac{z-1}{z+1}\right) = 0$, S.T the locus of z is $x^2+y^2 = 1$.

Soln:

Given $Re\left(\frac{z-1}{z+1}\right) = 0$, put $z = x+iy$

$Re\left(\frac{x+iy-1}{x+iy+}\right) = 0$

$Re\left(\frac{(x-1)+iy}{(x+1)+iy}\right) = 0 \qquad Re\left(\frac{a+ib}{c+id}\right) = \frac{ac+bd}{c^2+d^2}$

$\left(\frac{(x-1)(x+1)+y^2}{(x+1)^2+y^2}\right) = 0$

$x^2-1+y^2 = 0 \Rightarrow x^2+y^2 = 1$

4.If $z = x + iy$ is a complex number such that $arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{2}$, S.T the locus of z is $x^2+y^2 = 1$.

Soln:

Given $arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{2}$ put $z = x+iy$

$arg\left(\frac{x+iy-1}{x+iy+}\right)=\frac{\pi}{2}$

$arg\left(\frac{(x-1)+i}{(x+1)+iy}\right)=\frac{\pi}{2}$ $\quad$ $\mathbf{arg}\left(\frac{a+ib}{c+id}\right)=\tan^{-1}\left(\frac{bc-ad}{ac+bd}\right)$

$tan^{-1}\left(\frac{y(x+1)-y(x-1)}{(x-1)(x+1)+y^2}\right)=\frac{\pi}{2}$

$\left(\frac{y(x+1)-y(x-1)}{(x-1)(x+1)+y^2}\right)=tan\frac{\pi}{2}=\infty=\frac{1}{0}$

$(x-1)(x+1)+y^2=0$

$\mathbf{x^2-1+y^2=0 \;\Rightarrow\; x^2+y^2=1}$

5. If $z=x+iy$ is a complex number such that $arg\left(\frac{z-i}{z+2}\right)=\frac{\pi}{4}$, show that the locus of z is $x^2+y^2+3x-3y+2=0$.

Soln:

Given $arg\left(\frac{z-i}{z+2}\right)=\frac{\pi}{4}$ $\quad$ put $z=x+iy$

$arg\left(\frac{x+iy-i}{x+iy+2}\right)=\frac{\pi}{4}$

$arg\left(\frac{x+i(y-1)}{(x+2)+iy}\right)=\frac{\pi}{4}$ $\quad$ $\mathbf{arg}\left(\frac{a+ib}{c+id}\right)=\tan^{-1}\left(\frac{bc-ad}{ac+bd}\right)$

$tan^{-1}\left(\frac{(x+2)(y-1)-xy}{x(x+2)+y(y-1)}\right)=\frac{\pi}{4}$

$\left(\frac{(x+2)(y-1)-xy}{x(x+2)+y(y-1)}\right)=tan\frac{\pi}{4}=1$

$(x+2)(y-1)-xy=x(x+2)+y(y-1)$

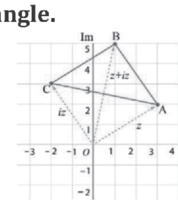
$\mathbf{x^2+y^2+3x-3y+2=0}$

Try yourself

If $z=x+iy$ is a complex number such that $arg\left(\frac{z-1}{z+1}\right)=\frac{\pi}{3}$, show that the locus of z is $\sqrt{3}x^2+\sqrt{3}y^2-2y-3=0$.

6.If $z=3+2i$, represent the complex numbers z, iz, and $z+iz$ in one Argand plane. S.t. these complex numbers form the vertices of an isosceles right triangle.

Soln:



Given, $z=3+2i$

Then $iz=i(3+2i)=3i-2=-2+3i$

$z+iz=1+5i$

Let $z_1=z=3+2i$, $\quad z_2=iz=-2+3i$,

$z_3=z+iz=1+5i$

$AB=|z_1-z_2|=|(3+2i)-(-2+3i)|$

$=|5-i|=\sqrt{(5)^2+(-1)^2}=\sqrt{\mathbf{26}}$

$BC=|z_2-z_3|=|(-2+3i)-(1+5i)|$

$=|-3-2i|=\sqrt{(-3)^2+(-2)^2}=\sqrt{\mathbf{13}}$

$CA=|z_3-z_1|=|(1+5i)-(3+2i)|$

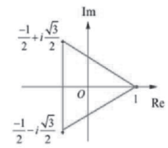
$=|-2+3i|=\sqrt{(-2)^2+(3)^2}=\sqrt{\mathbf{13}}$

$BC^2+CA^2=AB^2=26$

∴ Given complex numbers form the vertices of an isosceles right triangle.

7.Show that the points 1, $\frac{-1}{2}+i\frac{\sqrt{3}}{2}$ and $\frac{1}{2}-i\frac{\sqrt{3}}{2}$ are the vertices of an equilateral triangle.

Soln:



Let $z_1=1$ $\quad z_2=\frac{-1}{2}+i\frac{\sqrt{3}}{2}$ $\quad z_3=\frac{1}{2}-i\frac{\sqrt{3}}{2}$

$AB=|z_1-z_2|=\left|1-\left(\frac{-1}{2}+i\frac{\sqrt{3}}{2}\right)\right|=\sqrt{\mathbf{3}}$

$BC=|\mathbf{z_2-z_3}|$

$=\left|\left(\frac{-1}{2}+i\frac{\sqrt{3}}{2}\right)-\left(\frac{-1}{2}-i\frac{\sqrt{3}}{2}\right)\right|=|0+i\sqrt{3}|=\sqrt{3}$

$CA=|z_3-z_1|=\left|\left(\frac{-1}{2}-i\frac{\sqrt{3}}{2}\right)-1\right|=\left|\frac{-3}{2}-i\frac{\sqrt{3}}{2}\right|=\sqrt{3}$

AB=BC=CA

∴ Given points are the vertices of an equilateral triangle.

8.If z_1, z_2 and z_3 are three complex numbers S.T $|z_1|=1, |z_2|=2$, $|z_3|=3$ and $|z_1+z_2+z_3|=1$, show that $|9z_1z_2+4z_1z_3+z_yz_3|=6$.

Soln:

Given $|z_1|=1, |z_2|=2$, $|z_3|=3$ and $|z_1+z_2+z_3|=1$ $\quad$ ∵

$|z|^2=z\bar{z}$, $z_1\overline{z_1}=1, z_2\overline{z_2}=4$, $z_3\overline{z_3}=9$

$z_1=\frac{1}{\overline{z_1}}$, $z_2=\frac{4}{\overline{z_2}}$, $z_3=\frac{9}{\overline{z_3}}$

$|\mathbf{z_1+z_2+z_3}|=\left|\frac{1}{\overline{z_1}}+\frac{4}{\overline{z_2}}+\frac{9}{\overline{z_3}}\right|$

$\mathbf{1}=\frac{|z_2z_3+4z_1z_3+9z_1z_2|}{|z_1||z_2||z_3|}$

$|z_2z_3+4z_1z_3+9z_1z_2|=|z_1||z_2||z_3|$

$=\mathbf{1\times 2\times 3=6}$

9.If z_1, z_2 and z_3 are three complex number S.T. $|z_1|=|z_2|=|z_3|=r>0$ and $z_1+z_2+z_3\neq 0$. Prove that $\left|\frac{z_1z_2+z_2z_3+z_3z_1}{z_1+z_2+z_3}\right|=r$.

Soln:

Given $|z_1|=|z_2|=|z_3|=r$ $\quad$ ∵ $|z|^2=z\bar{z}$

$z_1\overline{z_1}=z_2\overline{z_2}=z_3\overline{z_3}=r^2$

$z_1=\frac{r^2}{\overline{z_1}}$, $z_2=\frac{r^2}{\overline{z_2}}$, $z_3=\frac{r^2}{\overline{z_3}}$

$|z_1+z_2+z_3|=\left|\frac{r^2}{\overline{z_1}}+\frac{r^2}{\overline{z_2}}+\frac{r^2}{\overline{z_3}}\right|$

$=r^2\left|\frac{\overline{z_1}\,\overline{z_2}+\overline{z_2}\,\overline{z_3}+\overline{z_3}\,\overline{z_1}}{\overline{z_1}\,\overline{z_2}\,\overline{z_3}}\right|$

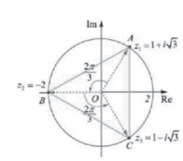
$|z_1+z_2+z_3|=r^2\frac{|z_1z_2+z_2z_3+z_1z_3|}{r^3}$

$=\frac{|z_1z_2+z_2z_3+z_1z_3|}{r}$

∴ $\left|\frac{z_1z_2+z_2z_3+z_3z_1}{z_1+z_2+z_3}\right|=r$

10.Suppose z_1, z_2 and z_3 are the vertices of an equilateral triangle inscribed in the circle $|z|=2$. If $z_1=1+i\sqrt{3}$, then find z_2 and z_3.

Soln:



Given, $|z|=r=2$ and

$z_1=1+i\sqrt{3}$;

$\theta=\alpha=tan^{-1}\left(\frac{\sqrt{3}}{1}\right)=\frac{\pi}{3}$

∴ Euler's form of $z_1=re^{i\theta}=2e^{i\frac{\pi}{3}}$

Clearly, z_2 is rotation of z_1 anti-clockwise by $\frac{2\pi}{3}$

MATHS

$z_2 = z_1 e^{i\frac{2\pi}{3}} = 2e^{i\frac{\pi}{3}} e^{i\frac{2\pi}{3}} = 2e^{i\pi} = -2$

Clearly, z_3 is rotation of z_2 anticlockwise by $\frac{2\pi}{3}$ $\quad$ $\boldsymbol{z_3 =}$

$\boldsymbol{z_2 e^{i\frac{2\pi}{3}} = -2\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) = 1 - i\sqrt{3}}$

Note :

$\boldsymbol{z = (1)^{\frac{1}{3}} = (1, \omega, \omega^2)}$

Here $\boldsymbol{\omega = \frac{-1}{2} + i\frac{\sqrt{3}}{2}, \omega^2 = \frac{-1}{2} - i\frac{\sqrt{3}}{2}}$

11.Solve the equation $z^3 + 8i = 0$, where $z \in C$.

Soln: $z^3 + 8i = 0$

$$z^3 = -8i$$

$$z^3 = (2i)^3 \times 1$$

$$z = 2i \times (1)^{\frac{1}{3}}$$

$$z = 2i(1, \omega, \omega^2)$$

$$z = 2i,\ 2i\left(\frac{-1}{2} + i\frac{\sqrt{3}}{2}\right),\ 2i\left(\frac{-1}{2} - i\frac{\sqrt{3}}{2}\right)$$

$$\boldsymbol{z = 2i, -i - \sqrt{3}, \quad -i + \sqrt{3}}$$

12.Solve the equation $z^3 + 27 = 0$, where $z \in C$

Soln: $z^3 + 27 = 0$

$$z^3 = -27 = -3 \times -3 \times -3$$

$$z^3 = (-3)^3 \times 1$$

$$z = -3 \times (1)^{\frac{1}{3}}$$

$$z = -3(1, \omega, \omega^2)$$

$$\boldsymbol{Z = -3, -3\omega, -3\omega^2}$$

$$\boldsymbol{z = -3, -3\left(\frac{-1}{2} + i\frac{\sqrt{3}}{2}\right), -3\left(\frac{-1}{2} - i\frac{\sqrt{3}}{2}\right)}$$

13.If $\omega \neq 1$ is a cube root of unity, show that the roots of the equation $(z-1)^3 + 8 = 0$ are $-1, 1-2\omega, 1-2\omega^2$.

Soln:

$$(z-1)^3 + 8 = 0$$

$$(z-1)^3 = -8 = (-2)^3 \times 1$$

$$(z-1) = -2 \times (1)^{\frac{1}{3}}$$

$$z - 1 = -2(1, \omega, \omega^2) = -2, -2\omega, -2\omega^2$$

$$\boldsymbol{Z = -1, 1 - 2\omega, 1 - 2\omega^2}$$

14.Find all the cube roots of $\sqrt{3} + i$

Soln:

Let $z^3 = re^{i\theta} \Rightarrow z = \left(re^{i\theta}\right)^{\frac{1}{3}}$

$$z = \left(\sqrt{3} + i\right)^{\frac{1}{3}}$$

$$r = \sqrt{\left(\sqrt{3}\right)^2 + (1)^2} = 2,$$

$$\theta = \alpha = tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$$

$$\sqrt{3} + i = 2cis\left(\frac{\pi}{6}\right)$$

$$\left(\sqrt{3} + i\right)^{\left(\frac{1}{3}\right)} = 2^{\frac{1}{3}} cis\left(\frac{(12k+1)\pi}{18}\right)$$

$k = 0,\quad z = 2^{\frac{1}{3}}\, cis(\frac{\pi}{18})$

$k = 1,\quad z = 2^{\frac{1}{3}}\, cis(\frac{13\pi}{18})$

$k = 2,\quad z = 2^{\frac{1}{3}}\, cis(\frac{25\pi}{18})$

15.If $\boldsymbol{\cos\alpha + \cos\beta + \cos\gamma = \sin\alpha + \sin\beta + \sin\gamma = 0}$, show that

(i)$\boldsymbol{\cos 3\alpha + \cos 3\beta + \cos 3\gamma = 3\cos(\alpha + \beta + \gamma)}$

(ii)$\boldsymbol{\sin 3\alpha + \sin 3\beta + \sin 3\gamma = 3\sin(\alpha + \beta + \gamma)}$.

Soln:

$a = \cos\alpha + i\sin\alpha,\ b = \cos\beta + i\sin\beta,\ c = \cos\gamma + i\sin\gamma$

$\boldsymbol{if\ a + b + c = 0\ \text{then}\ a^3 + b^3 + c^3 = 3abc}$

$$(\cos\alpha + i\sin\alpha)^3 + (\cos\beta + i\sin\beta)^3(\cos\gamma + i\sin\gamma)^3$$
$$= 3(\cos\alpha + i\sin\alpha)(\cos\beta + i\sin\beta)(\cos\gamma + i\sin\gamma)$$

$$(\cos 3\alpha + \cos 3\beta + \cos 3\gamma) + i(\sin 3\alpha + \sin 3\beta + \sin 3\gamma)$$
$$= 3[\cos\ (\alpha + \beta + \gamma) + i\sin(\alpha + \beta + \gamma)]$$

$(i)\ \cos 3\alpha + \cos 3\beta + \cos 3\gamma = 3\cos(\alpha + \beta + \gamma)$

$(ii)\ \sin 3\alpha + \sin 3\beta + \sin 3\gamma = 3\sin(\alpha + \beta + \gamma)$

16.If $\boldsymbol{2\cos\alpha = x + \frac{1}{x}}$ and $\boldsymbol{2\cos\beta = y + \frac{1}{y}}$, show that

(i)$\boldsymbol{\frac{x}{y} + \frac{y}{x} = 2\cos(\alpha - \beta)}$

(ii) $\boldsymbol{xy - \frac{1}{xy} = 2i\sin(\alpha + \beta)}$

(iii)$\boldsymbol{\frac{x^m}{y^n} - \frac{y^n}{x^m} = 2i\sin(m\alpha - n\beta)}$

(iv)$\boldsymbol{x^m y^n + \frac{1}{x^m y^n} = 2\cos(m\alpha + n\beta)}$

Soln:

Given $x + \frac{1}{x} = 2cos\alpha$

Let $x = cos\alpha + isin\alpha$,

similarly $y = cos\beta + isin\beta$

(i) $\frac{x}{y} = cos(\alpha - \beta) + isin(\alpha - \beta)$

$$\frac{y}{x} = cos(\alpha - \beta) - isin(\alpha - \beta)$$

$$\frac{x}{y} + \frac{y}{x} = 2cos(\alpha - \beta)$$

(ii) $xy = cos(\alpha + \beta) + isin(\alpha + \beta)$

$$\frac{1}{xy} = cos(\alpha + \beta) - isin(\alpha + \beta)$$

$xy-\frac{1}{xy}=2isin(\alpha+\beta)$

(iii) $\frac{x^m}{y^n}=cos(m\alpha-n\beta)+isin(m\alpha-n\beta)$

$\frac{y^n}{x^m}=cos(m\alpha-n\beta)-isin(m\alpha-n\beta)$

$\frac{x^m}{y^n}-\frac{y^n}{x^m}=2isin(m\alpha-n\beta)$

(iv)$x^my^n=cos(m\alpha+n\beta)+isin(m\alpha+n\beta)$

$\frac{1}{x^my^n}=cos(m\alpha+n\beta)-isin(m\alpha+n\beta)$

$x^my^n+\frac{1}{x^my^n}=2cos(m\alpha+n\beta)$

2 ,3 Mark

1.Evaluate

Soln:

(i) $i^{1729}=i$

(ii) $i^{-1924}+i^{2018}=i^0+i^2=1-1=0;$

(iii) $i^{59}+\frac{1}{i^{59}}=i^{59}-i^{59}=0$

(iv) $ii^2i^3\ldots..i^{40}=i^{1+2+3+\cdots+40}$

$=i^{\left(\frac{40\times41}{2}\right)}=i^{820}=1$

2. If $z_1=6+7i,\quad z_2=3-5i$

Soln:

$z_1+z_2=(6+3)+i(7-5)=9+2i$

$z_1-z_2=(6-3)+i(7+5)=3+12i$

$z_1z_2=(6+7i)(3-5i)\ =18-30i+21i-35(-1)$

$=53-9i$

$\frac{z_1}{z_2}=\frac{6+7i}{3-5i}=\frac{-17+5}{34}=\frac{-17}{34}+\frac{51}{34}\quad \frac{a+ib}{c+id}=\frac{(ac+bd)+i\,(bc-ad)}{c^2+d^2}$

3.Show that (i) $\left(\frac{19-7i}{9+i}\right)^{12}+\left(\frac{20-5i}{7-6i}\right)^{12}$ is real

Soln :

$\frac{19-7i}{9+i}=2-i\ ,\ \frac{20-5}{7-6i}=2+i$

$$z=\left(\frac{19-7i}{9+i}\right)^{12}+\left(\frac{20-5i}{7-6i}\right)^{12}$$

$z=(2-i)^{12}+(2+i)^{12}$

$\bar{z}=(2+i)^{12}+(2-i)^{12}$

$\bar{z}=z$, z is real

4.Show that (i) $\left(\frac{19+9i}{5-3i}\right)^{15}-\left(\frac{8+i}{1+2i}\right)^{15}$ is purely imaginary

Soln:

$\frac{19+9i}{5-3i}=2+3i,\frac{8+i}{1+2i}=2-3i$

$z=\left(\frac{19+9i}{5-3i}\right)^{15}-\left(\frac{8+i}{1+2i}\right)^{15}$

$z=(2+3i)^{15}-(2-3i)^{15}$

$\bar{z}=(2-3i)^{15}-(2+3i)^{15}$

$\bar{z}=-z$

$\therefore z$ is purely imaginary.

5. If $z=3+4i$, then find z^{-1}

Soln:

$z^{-1}=\frac{1}{z}=\frac{1}{3+4i}=\frac{3-4i}{3^2+4^2}=\frac{3-4i}{25}$

$=\frac{3}{25}+\frac{-4i}{25}\qquad \frac{1}{a+ib}=\frac{(a-ib)}{a^2+b^2}$

6. If $z=(2+3i)(1-i)$, then find z^{-1}

Soln:

$z=2-2i+3i+3i(-i)=2+i-3=-1+i$

$$z^{-1}=\frac{1}{z}=\frac{1}{-1+i}=\frac{-1-i}{(-1)^2+1^2}$$

$$=\frac{-1-i}{2}=\frac{-1}{2}-\frac{i}{2}$$

7.If $z_1=3,\quad z_2=-7i,\ z_3=5+4i$ show that

$z_1(z_2+z_3)=z_1z_2+z_1z_3$

Soln:

$z_2+z_3=-7i+(5+4i)=5-3i$

$z_1(z_2+z_3)=3(5-3i)=15-9i -------\to(1)$

$z_1z_2+z_1z_3=3(-7i)+3(5+4i)=-21i+15+12i$

$=15-9i ----------\to(2)$

$(1),(2)\Rightarrow z_1(z_2+z_3)=z_1z_2+z_1z_3$

8.Which one of the point $i,-2+i$ and 3 is farthest and shortest from the origin?

Soln :

Let $z_1=i,\quad z_2=-2+i,\ \ z_3=3$

$|z_1|=|i|=\sqrt{1^2}=1$

$|z_2|=|-2+i|=\sqrt{(-2)^2+1^2}=\sqrt{5}$

$|z_3|=|3|=3$Farthest point is 3 and shortest point is i

9. Which one of the point $10-8i$, $11+6i$ is closest to $1+i$.

Soln :

Let $z_1=10-8i,\quad z_2=11+6i,\ $ and $z=1+i$

$|z_1-z|=|(10-8i)-(1+i)|=|9-9i|=\sqrt{9^2+(-9)^2}$

$=\sqrt{162}$

$|z_2-z|=|(11+6i)-(1+i)|=|10+5i|=\sqrt{10^2+5^2}$

$=\sqrt{125}$

$11+6i$ is closest to $1+i$.

9. If $(1+i)(1+2i)(1+3i)\ldots\ldots..(1+ni)=x+iy$ then show that $2.5.10\ldots..(1+n^2)=x^2+y^2$.

Soln:

$|(1+i)(1+2i)(1+3i)\ldots\ldots..(1+ni)|=|x+iy|$

$|(1+i)||(1+2i)||(1+3i)|\ldots\ldots..|(1+ni)|=|x+iy|$

$(\sqrt{1^2+1^2})(\sqrt{1^2+2^2})(\sqrt{1^2+3^2})\ldots\ldots..(\sqrt{1^2+n^2})=\sqrt{x^2+y^2}$

$(\sqrt{2})(\sqrt{5})(\sqrt{10})\ldots\ldots..(\sqrt{1^2+n^2})=\sqrt{x^2+y^2}$

Taking square on both sides

$2.5.10\ldots..(1+n^2)=x^2+y^2$

MATHS

Square root of a complex number

If $z = x \pm iy$, then

$$\sqrt{z} = \sqrt{x \pm iy} = \pm\left(\sqrt{\frac{|z|+x}{2}} \pm i\sqrt{\frac{|z|-x}{2}}\right)$$

10. Find the square root of a complex number $6-8i, 4+3i$.

Soln:

$|6-8i| = \sqrt{(6)^2+(-8)^2} = \sqrt{100}$

$|z| = 10$

$\sqrt{6-8i} = \pm\left(\sqrt{\frac{10+6}{2}} - i\sqrt{\frac{10-6}{2}}\right)$

$= \pm\left(\sqrt{\frac{16}{2}} - i\sqrt{\frac{4}{2}}\right)$

$= \pm(\sqrt{8} - i\sqrt{2})$

$= \pm(2\sqrt{2} - i\sqrt{2})$

$|4+3i| = \sqrt{4^2+3^2} = \sqrt{25}$

$|z| = 5$

$\sqrt{4+3i} = \pm\left(\sqrt{\frac{5+4}{2}} + i\sqrt{\frac{5-4}{2}}\right)$

$= \pm\left(\sqrt{\frac{9}{2}} + i\sqrt{\frac{1}{2}}\right)$

$= \pm\left(\frac{3}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right)$

Try yourself :

Find the square root of a complex number

$-6+8i, -5-12i$

11. If area of triangle formed by z, iz, z+iz is 50 sq.unit. find the value of $|z|$.

Soln : Area of triangle $= \frac{1}{2}|z|^2 = 50$

$|z|^2 = 100 \Rightarrow |z| = 10$

12. If $|z| = 2$ show that $3 \le |z+3+4i| \le 7$

Soln:

$||z| - |3+4i|| \le |z+3+4i| \le |z| + |3-4i|$

$|2-5| \le |z+3+4i| \le 2+5$

$|-3| \le |z+3+4i| \le 7$

$3 \le |z+3+4i| \le 7$

13. Show that $\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3 = -2i$

Soln

$\frac{1+i}{1-i} = i$, $\frac{1-i}{1+i} = -i$

$\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3 = i^3 - (-i)^3 = -i - i = -2i$

14.Simplify $\left[\frac{1+cos2\theta+isin2\theta}{1+cos2\theta-isin2\theta}\right]^{30}$

Soln

$\left[\frac{1+cos2\theta+is}{1+cos2\theta-is}\right] = cos2\theta + isin2\theta$

$\left[\frac{1+cos2\theta+is}{1+cos2\theta-isin}\right]^{30} = (cos2\theta + isin2\theta)^{30}$

$= (cos60\theta + isin60\theta)$

15. Find the locus of z If $|z+i| = |z-1|$

Soln:

$|z+i| = |z-1|$

$|x+iy+i| = |x+iy-1|$

$|x+i(y+1)| = |(x-1)+iy|$

$\sqrt{x^2+(y+1)^2} = \sqrt{(x-1)^2+y^2}$

$x^2+(y+1)^2 = (x-1)^2+y^2$

$x^2+y^2+2y+1 = x^2-2x+1+y^2$

$2x+2y = 0$

$x+y = 0$

16. write $\overline{3i} + \frac{1}{2-i}$ in rectangular form.

Soln:

$\overline{3i} + \frac{1}{2-i} = -3i + \frac{2+i}{5}$ $\qquad \frac{1}{a+ib} = \frac{(a-ib)}{a^2+b^2}$

$= \frac{2-14i}{5} = \frac{2}{5} - \frac{14i}{5}$

17.Find $\left|\frac{i(2+i)^3}{(1+i)^2}\right|$

Soln:

$\left|\frac{i(2+i)^3}{(1+i)^2}\right| = \frac{1\left(\sqrt{2^2+1^2}\right)^3}{\left(\sqrt{1^2+1^2}\right)^2} = \frac{(\sqrt{5})^3}{(\sqrt{2})^2} = \frac{5\sqrt{5}}{2}$

18.The complex numbers u,v and w are related by $\frac{1}{u} = \frac{1}{v} + \frac{1}{w}$. If $v = 3-4i$ and $w = 4+3i$, Find u in rectangular form.

Soln:

$\frac{1}{v} = \frac{1}{3-4i} = \frac{3+4i}{25}$ $\qquad$ *hint:* $\frac{1}{a+ib} = \frac{(a-ib)}{a^2+b^2}$

$\frac{1}{w} = \frac{1}{4+3i} = \frac{4-3i}{25}$

$\frac{1}{u} = \frac{1}{v} + \frac{1}{w} = \frac{7+i}{25}$

$u = \frac{25}{7+i} = \frac{25(7-i)}{50} = \frac{7}{2} - \frac{i}{2}$

19.Show that $|3z-5+i| = 4$ represents a circle, then find its centre and radius.

Soln:

$|3z-(5-i)| = 4$

$\left|z - \left(\frac{5}{3} - \frac{i}{3}\right)\right| = \frac{4}{3}$

centre $\left(\frac{5}{3}, -\frac{1}{3}\right)$ radius $\frac{4}{3}$

Hint: $|z - z_0| = r$

Try yourself

(i) $|z+2-i|<2$, (ii) $|z-2-i|=3$

(iii) $|2z+2-4i|=2$ (iv) $|3z-6+12i|=8$

20. If $\omega \neq 1$ is a cube root of unity ,

show that $\frac{a+b\omega+c\omega^2}{b+c\omega+a\omega^2}+\frac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2}=-1$

Soln:

$\frac{a+b\omega+c\omega^2}{b+c\omega+a\ \ 2}\times\frac{\omega}{\omega}+\frac{a+b\omega+c\ \ 2}{c+a\omega+b\omega^2}\times\frac{\omega^2}{\omega^2}$

$$=\frac{\omega(a+b\omega+c\omega^2)}{a+b\omega+c\omega^2}+\frac{\omega^2(a+b\omega+c\omega^2)}{a+b\omega+c\omega^2}$$

$=\omega+\omega^2=-1$

21.Find the fourth roots of unity.

Soln:

Given $z^4=1$

$(z^2)^2=1$

$z^2=\pm\sqrt{1}$

$z^2=\pm 1$

$z^2=1$ $\qquad z^2=-1$

$z=\pm\sqrt{1}$ $\qquad z=\pm\sqrt{-1}$

$\boldsymbol{z=\pm 1}$ $\qquad \boldsymbol{z=\pm i}$

22.Find the cube roots of unity.

Soln:

Given $z^3=1$

$z^3-1=0$

$(z-1)(z^2+z+1)=0$

$z-1=0$ $\qquad z^2+z+1=0$ $\qquad \boldsymbol{z=1}$

$\boldsymbol{z=\frac{-1\pm i\sqrt{3}}{2}}$

23.Show that $\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)^5+\left(\frac{\sqrt{3}}{2}-\frac{i}{2}\right)^5=-\sqrt{3}$

Soln:

$\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)^5+\left(\frac{\sqrt{3}}{2}-\frac{i}{2}\right)^5=(-i\omega)^5+(i\omega^2)^5=-i\omega^2+i\omega$

$=-i\left(-\frac{1}{2}-i\frac{\sqrt{3}}{2}\right)+i\left(-\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)$

$=-\sqrt{3}$

22. Evaluate $\sum_{k=1}^{18}(cos\frac{2k\pi}{9}+isin\frac{2k\pi}{9})$

Soln:

$$\sum_{k=0}^{8}\left(cos\frac{2k\pi}{9}+isin\frac{2k\pi}{9}\right)=0$$

$1+\sum_{k=1}^{8}\left(cos\frac{2k\pi}{9}+isin\frac{2k\pi}{9}\right)=0$

$\sum_{k=1}^{8}\left(cos\frac{2k\pi}{9}+isin\frac{2k\pi}{9}\right)=-1$

23. If $\omega \neq 1$ is a cube root of unity, then show that the following

(i) $(1-\omega+\omega^2)^6+(1+\omega-\omega^2)^6=128$

(ii) $(1+\omega)(1+\omega^2)(1+\omega^4)\ldots\ldots\left(1+\omega^{2^{11}}\right)=1$

Soln:

(i) $\qquad (1-\omega+\omega^2)^6+(1+\omega-\omega^2)^6$

$=(-\omega-\omega)^6+(-\omega^2-\omega^2)^6$

$=(-2\omega)^6+(-2\omega^2)^6$

$=64+64=128$

(ii) $(1+\omega)(1+\omega^2)(1+\omega^4)(1+\omega^8)\ldots\ldots(1+\omega^{2^{11}})$

$=[(-\omega^2)(-\omega)][(-\omega^2)(-\omega)]\ldots upto\ 6\ times$

$=1\times1\times1\times1\times1\times1=1$

24.State and prove Triangle inequality

Triangle inequality $|z_1+z_2|\leq|z_1|+|z_2|$

Proof:

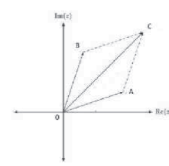
$OA=|z_1|, OB=|z_2|, OC=|z_1+z_2|$

In Δ OAC, $\qquad OC<OA+AC$

$|z_1+z_2|<|z_1|+|z_2|$ – – – – – (1)

Suppose the points are in collinear

$|z_1+z_2|=|z_1|+|z_2|$ – – – – → (2)



From (1),(2)

$$|z_1+z_2|\leq|z_1|+|z_2|$$

DISCRETE MATHEMATICS

5 MARK

Important hints:

Let * be a binary operation on S

i) Closure property : $\forall a,b\in S \Rightarrow a*b\in S$

ii) Commutative property : $a*b=b*a$, $\forall a,b\in S$

iii) Associative property :

$$a*(b*c)=(a*b)*c, \forall a,b,c\in S$$

iv) Existence of identity : $a*e=e*a=a$, e is the identity element, $e\in S, \forall a\in S$

v) Existence of inverse : a^{-1} is the inverse of a

$$a*\quad a^{-1}=a^{-1}*a=e,\quad a^{-1}\in S$$

1.Verify closure, commutative, associative, existence of identity, and existence of inverse for

$m*n=m+n-mn$, $m,n\epsilon Z$

Soln:

<u>Closure property</u>:

$m,n\epsilon Z$, clearly $m+n-mn\ \epsilon Z$

∴ closure property true

<u>Associative property</u>:

$(l*m)*n=l*(m*n)$

$(l*m)*n=l+m+n-lm-mn-nl+lmn$

$= l * (m * n)$

∴ associative property true

Identity property:

$m * e = e * m = m$

$m + e - me = m$

$e = 0 \epsilon Z$

∴ identity property true

Inverse property:

$m * m^{-1} = m^{-1} * m = e = 0$

$m^{-1} = \frac{-m}{1-m} \notin z$

∴ inverse property not true

Commutative property:

$m * n = n * m = m + n - mn = n + m - nm$

∴ commutative property true

2.Let A be $Q \setminus \{1\}$. Define * on A by $x * y = x + y - xy$, Verify closure, commutative, associative, existence of identity, and existence of inverse properties satisfied by * on A.

Soln:

Closure property:

$x, y \epsilon Q \setminus \{1\}, \quad x \neq 1, \; y \neq 1$

$\Rightarrow \; x + y - xy \neq 1$

$x * y \epsilon Q \setminus \{1\}$ ∴ closure property true

Associative property:

$(x * y) * z = x * (y * z)$

∴ associative property true

Identity property:

$x * e = e * x = x$

$e = 0 \; \epsilon Q \setminus \{1\}$

∴ identity property true

Inverse property:

$x * x^{-1} = x^{-1} * x = e = 0$

$x^{-1} = \frac{-x}{1-x} \epsilon Q \setminus \{1\}$

∴ inverse property true

Commutative property:

$x * y = x + y - xy = y + x - yx = y * x$

∴ commutative property true

3.Verify closure, commutative , associative, existence of identity, and inverse for $M = \left\{\begin{pmatrix} x & x \\ x & x \end{pmatrix} \in R - \{0\}\right\}$. * be matrix multiplication

Soln:

Let $*$be the matrix multiplication.

Closure property:

Let, $A = \begin{pmatrix} x & x \\ x & x \end{pmatrix}, B = \begin{pmatrix} y & y \\ y & y \end{pmatrix} \; \because \; x, y \neq 0 \Rightarrow 2xy \neq 0$

$AB = \begin{pmatrix} 2xy & 2xy \\ 2xy & 2xy \end{pmatrix} \epsilon M$

∴ closure property true.

Commutative property :

$AB = \begin{pmatrix} 2xy & 2xy \\ 2xy & 2xy \end{pmatrix}, BA = \begin{pmatrix} 2yx & 2yx \\ 2yx & 2yx \end{pmatrix}$

$AB = BA$

∴ commutative property true.

Associative property:

Matrix multiplication always satisfies associative property

Existence of identity property: $A * E = E * A = A$

$\begin{pmatrix} x & x \\ x & x \end{pmatrix}\begin{pmatrix} e & e \\ e & e \end{pmatrix} = \begin{pmatrix} x & x \\ x & x \end{pmatrix}$

$2ex = x$

$e = \frac{1}{2} \qquad \therefore \; E = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} \epsilon M$

∴ identity property true

Existence of inverse property:

$A * A^{-1} = A^{-1} * A = E = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

$\begin{pmatrix} x & x \\ x & x \end{pmatrix}\begin{pmatrix} a & a \\ a & a \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

$2ax = \frac{1}{2} \quad \Rightarrow a = \frac{1}{4x}$

$\therefore \; A^{-1} = \begin{pmatrix} \frac{1}{4x} & \frac{1}{4x} \\ \frac{1}{4x} & \frac{1}{4x} \end{pmatrix} \epsilon M$

∴ inverse property true

4.Verify closure property, commutative property, associative property ,existence of identity, and existence of inverse for the operation $\times_{11}$ on a subset A = {1,3,4,5,9}of the set of remainders {0,1,2,3,4,5,6,7,8,9,10}.

Soln:

$\times_{11}$	1	3	4	5	9
1	1	3	4	5	9
3	3	9	1	4	5
4	4	1	5	9	3
5	5	4	9	3	1
9	9	5	3	1	4

Closure property:

From the table closure property true.

Commutative property:

From the table commutative property true.

Associative property:

$\times_{11}$ always satisfies associative property .

Identity property:

Identity element $1 \epsilon A$

∴ identity property true

Inverse property:

Inverse element of 1,3,4,5 and 9 are 1,4,3,9 and 5 respectively. ∴ inverse property true

5.Verify closure property, commutative property, associative property,existence of identity, and existence of inverse for the operation $+_5$ on Z_5 using table corresponding to addition modulo 5.
Soln:

$Z_5 = \{0,1,2,3,4\}$

$+_5$	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

Closure property:

From the table closure property true

Commutative property:

From the table commutative property true

Associative property:

$+_5$ always satisfies associative property.

Identity property:

identity element $0 \epsilon Z_5$

∴ identity property true.

Inverse property:

Inverse element of 0,1,2,3 and 4 are 0,4,3,2 and 1 respectively.

∴ inverse property true

6.Verify closure, commutative, associative, identity, and inverse property for
$\boldsymbol{a * b = \frac{a+b}{2} \forall\ a, b \in Q}$
Soln:

Closure property:

Clearly $a, b \in Q \implies \frac{a+b}{2} \in Q$

∴ closure property true

Associative property:

$(a * b) * c = \frac{a+b+2c}{4}$

$a * (b * c) = \frac{2a+b+c}{4}$

$(a * b) * c \neq a * (b * c)$

∴ Associative property is not true

Identity property:

$a * e = e * a = a$

$a * e = a$

$\frac{a+e}{2} = \boldsymbol{a}$

$e = a$

Uniqueness of identity is not preserved
∴ identity property is not true
Inverse property:

∴ inverse property is not true

Commutative property:

$a * b = b * a = \frac{a+b}{2}$

∴commutative property true

2,3 Marks

1.In an algebraic structure the identity element must be unique
Soln:

Let e_1 and e_2 be the identity element of S

$$a * e_1 = e_1 * a = a \qquad \forall a \in S$$

$$a * e_2 = e_2 * a = a \qquad \forall a \in S$$

$$a * e_1 = a * e_2$$

$$\therefore e_1 = e_2$$

2.In an algebraic structure the inverse element must be unique
Soln:

Let a_1 and a_2 be the inverse element of a in S

$$a * a_1 = a_1 * a = e \qquad \forall a \in S$$

$$a * a_2 = a_2 * a = e \qquad \forall a \in S$$

$$a * a_1 = a * a_2$$

$$a_1 = a_2$$

3. Let $A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ be any two Boolean matrices of the same type. Find $A \vee B$ and $A \wedge B$.
Soln:

$A \vee B = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \vee \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$A \wedge B = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \wedge \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$

***Try Yourself*:**

Let $A = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{pmatrix}, B = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix},$

$C = \begin{pmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix}$ be any three Boolean matrices of the same type. Find (i) $A \vee B$ (ii) $A \wedge B$ (iii) $(A \vee B) \wedge C$ (iv) $(A \wedge B) \vee C$.

4.Show that $\boldsymbol{p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)}$

p	q	$p \leftrightarrow q$	$p \to q$	$q \to p$	$(p \to q) \wedge (q \to p)$
T	T	T	T	T	T
T	F	F	F	T	F
F	T	F	T	F	F
F	F	T	T	T	T

$$p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$$

MATHS

MATHS

5. Show that $p \leftrightarrow q \equiv (p \wedge q) \vee (\neg q \wedge \neg p)$

p	q	$p \leftrightarrow q$	$\neg p$	$\neg q$	$p \wedge q$	$\neg q \wedge \neg p$	$(p \wedge q) \vee (\neg q \wedge \neg p)$
T	T	T	F	F	T	F	T
T	F	F	F	T	F	F	F
F	T	F	T	F	F	F	F
F	F	T	T	T	F	T	T

$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg q \wedge \neg p)$

6. Show that $p \rightarrow (\neg q \vee r) \equiv \neg p \vee (\neg q \vee r)$

P	q	r	$\neg p$	$\neg q$	$\neg q \vee r$	$p \rightarrow (\neg q \vee r)$	$\neg p \vee (\neg q \vee r)$
T	T	T	F	F	T	T	T
T	T	F	F	F	F	F	F
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	F	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

$p \rightarrow (\neg q \vee r) \equiv \neg p \vee (\neg q \vee r)$

7. Show that $p \rightarrow (q \rightarrow r) \equiv (p \wedge q) \rightarrow r$

p	q	r	$q \rightarrow r$	$p \rightarrow (q \rightarrow r)$	$p \wedge q$	$(p \wedge q) \rightarrow r$
T	T	T	T	T	T	T
T	T	F	F	F	T	F
T	F	T	T	T	F	T
T	F	F	T	T	F	T
F	T	T	T	T	F	T
F	T	F	F	T	F	T
F	F	T	T	T	F	T
F	F	F	T	T	F	T

$p \rightarrow (q \rightarrow r) \equiv (p \wedge q) \rightarrow r$

8.Using truth table whether the statements $\neg(p \vee q) \vee (\neg p \wedge q)$ *and* $\neg p$ are logically equivalent.

p	q	$\neg p$	$p \vee q$	$\neg(p \vee q)$	$\neg p \wedge q$	$\neg(p \vee q) \vee (\neg p \wedge q)$
T	T	F	T	F	F	F
T	F	F	T	F	F	F
F	T	T	T	F	T	T
F	F	T	F	T	F	T

$\neg(p \vee q) \vee (\neg p \wedge q)$ *and* $\neg p$ are logically equivalent.

9.Show that $\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

p	q	$p \leftrightarrow q$	$\neg(p \leftrightarrow q)$	$\neg q$	$p \leftrightarrow \neg q$
T	T	T	F	F	F
T	F	F	T	T	T
F	T	F	T	F	T
F	F	T	F	T	F

$\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

10. Show that $p \rightarrow q$ *and* $q \rightarrow p$ are not equivalent

P	q	$p \rightarrow q$	$q \rightarrow p$
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T

$p \rightarrow q$ *and* $q \rightarrow p$ are not equivalent

11. Show that $q \rightarrow p \equiv \neg p \rightarrow \neg q$

P	q	$q \rightarrow p$	$\neg p$	$\neg q$	$\neg p \rightarrow \neg q$
T	T	T	F	F	T
T	F	T	F	T	T
F	T	F	T	F	F
F	F	T	T	T	T

$q \rightarrow p \equiv \neg p \rightarrow \neg q$

12.Prove that $\neg(p \wedge q) \equiv \neg p \vee \neg q$

P	q	$p \wedge q$	$\neg(p \wedge q)$	$\neg p$	$\neg q$	$\neg p \vee \neg q$
T	T	T	F	F	F	F
T	F	F	T	F	T	T
F	T	F	T	T	F	T
F	F	F	T	T	T	T

$(p \wedge q) \equiv \neg p \vee \neg q$

13.Verify whether the compound propositions are tautology or contradiction . $((p \vee q) \wedge \neg p) \rightarrow q$

P	q	$p \vee q$	$\neg p$	$(p \vee q) \wedge \neg p$	$((p \vee q) \wedge \neg p) \rightarrow q$
T	T	T	F	F	T
T	F	T	F	F	T
F	T	T	T	T	T
F	F	F	T	F	T

$((p \vee q) \wedge \neg p) \rightarrow q$ is a tautology.

14.Construct the truth table for $(p\bar{\vee}q) \wedge (p\bar{\vee}\neg q)$

P	q	$p\bar{\vee}q$	$\neg q$	$p\bar{\vee}\neg q$	$(p\bar{\vee}q) \wedge (p\bar{\vee}\neg q)$
T	T	F	F	T	F
T	F	T	T	F	F
F	T	T	F	F	F
F	F	F	T	T	F

15. show that without using truth table

$\boldsymbol{p \to (q \to r) \equiv (p \wedge q) \to r}$

Soln: $p \to (q \to r) \equiv \neg p \vee (q \to r)$

$\equiv \neg p \vee (\neg q \vee r)$

$\equiv (\neg p \vee \neg q) \vee r$

$\equiv \neg(p \wedge q) \vee r$

$\equiv (p \wedge q) \to r$

$p \to (q \to r) \equiv (p \wedge q) \to r$

16.Show that without using truth table

$\boldsymbol{p \leftrightarrow q \equiv (p \wedge q) \vee (\neg q \wedge \neg p)}$

Soln:

$p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$

$\equiv (\neg p \vee q) \wedge (p \vee \neg q)$

$\equiv (\neg p \wedge (p \vee \neg q) \vee ([q \wedge (p \vee \neg q)])$

$\equiv [(\neg p \wedge p) \vee (\neg p \wedge \neg q)] \vee [(q \wedge p) \vee (q \wedge \neg q)]$

$\equiv [F \vee (\neg p \wedge \neg q)] \vee [(q \wedge p) \vee F]$

$\equiv (q \wedge p) \vee (\neg p \wedge \neg q)$

$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg q \wedge \neg p)$

17. check whether the $\boldsymbol{p \to (q \to p)}$ is tautology or contradiction without using truth table.

Soln:

$p \to (q \to p) \equiv \neg p \vee (q \to p)$

$\equiv \neg p \vee (\neg q \vee p)$

$\equiv \neg p \vee (p \vee \neg q)$

$\equiv (\neg p \vee p) \vee \neg q$

$\equiv T \vee \neg q$

$\equiv T \therefore$ $\boldsymbol{p \to (q \to p)}$ **is tautology**

DIFFERENTIALS AND PARTIAL DERIVATIVES

Important hints:

linear approximation :

$$L(x) = f(x_0) + f'(x_0)(x - x_0)$$

Euler theorem : $\displaystyle x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = nf$
$Degree = n = N.degree - D.degree$

1.Find the linear approximation for f(x)= $\sqrt{1+x}, x \geq -1, at\ x_0 = 3$ Use the linear approximation to estimate $f(3.2)$

Soln

$f(x) = \sqrt{1+x}$, $x_0 = 3, \Delta x = 0.2$ and

hence $f(3) = \sqrt{1+3} = 2.$

$$f'(x) = \frac{1}{2\sqrt{1+x}} \Rightarrow f'(3) = \frac{1}{2\sqrt{1+3}} = \frac{1}{4}$$

$L(x) = f(x_0) + f'(x_0)(x - x_0)$

$= 2 + \frac{1}{4}(x-3) = \frac{x}{4} + \frac{5}{4}$

$$f(3.2) = \sqrt{4.2} \cong L(3.2) = \frac{3.2}{4} + \frac{5}{4} = 2.050$$

2.Use linear approximation to find an approximate value of $\sqrt{9.2}$ without using a calculator

Soln :

$f(x) = \sqrt{x}, x_0 = 9, \Delta x = 0.2$

$f(9) = 3,$

$f'(x) = \frac{1}{2\sqrt{x}}$, $f'(x) = \frac{1}{2\sqrt{9}} = \frac{1}{(2\times3)} = \frac{1}{6}$

$L(x) = f(x_0) + f'(x_0)(x - x_0)$

$\sqrt{9.2} = f(9) + f'(9)(x - 9)$

$$= 3 + \frac{1}{6}(9.2 - 9) = 3 + \frac{0.2}{6} = 3.0333$$

3.*If $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, show that*

$$\boldsymbol{x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u}$$

Soln:

$$u = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$$

$$f = \sin u = \left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$$

$Degree = n = N.degree - D.degree$

$n = 1 - \frac{1}{2} = \frac{1}{2}$

$\therefore f(x,y) is\ a\ homgeneous\ function\ of\ degree\ is$

$$n = \frac{1}{2}$$

By Euler theorem, $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = nf$

$$x\frac{\partial}{\partial x}(\sin u) + y\frac{\partial}{\partial y}(\sin u) = \frac{1}{2}\sin u$$

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u$$

4.*If* $\boldsymbol{u(x,y) = \frac{x^2+y^2}{\sqrt{x+y}}}$, ***prove that*** $\boldsymbol{x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{3}{2}u}$.

Soln:

$$u(x,y) = \frac{x^2+y^2}{\sqrt{x+y}}$$

MATHS

$Degree = n = N.degree - D.degree$

$$n = 2 - \frac{1}{2} = \frac{3}{2}$$

$\therefore u(x, y) is\ a\ homgeneous\ function\ of\ degree\ is$

$$n = \frac{3}{2}$$

$By\ Euler\ theorem,\quad x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = \text{n}f$

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{3}{2}u$$

5. $\boldsymbol{If\ v(x, y) = \log\left(\frac{x^2 + y^2}{x + y}\right), prove\ that\ x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y} = 1}$

Soln

$v(x, y) = \log\left(\frac{x^2+y^2}{x+y}\right)$

$f = e^v = \dfrac{x^2 + y^2}{x + y}$

$Degree = n = N.degree - D.degree$

$$n = 2 - 1 = 1$$

$\therefore f(x, y) is\ a\ homgeneous\ function\ of\ degree\ is$

$$n = 1$$

$By\ Euler\ theorem,\quad x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = \text{n}f$

$$x\frac{\partial e^v}{\partial x} + y\frac{\partial e^v}{\partial y} = (1)e^v$$

$$x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y} = 1$$

6.If w(x,y,z)= $\boldsymbol{\log\left(\frac{5x^3y^4+7y^2xz^4-75y^3z^4}{x^2+y^2}\right), find}$
$\boldsymbol{x\frac{\partial w}{\partial x} + y\frac{\partial w}{\partial y} + z\frac{\partial w}{\partial z}}$

Soln

$w(x, y, z) = \log\left(\dfrac{5x^3y^4 + 7y^2xz^4 - 75y^3z^4}{x^2 + y^2}\right)$

$$f = e^w = \left(\frac{5x^3y^4 + 7y^2xz^4 - 75y^3z^4}{x^2 + y^2}\right)$$

$Degree = n = N.degree - D.degree$

$$n = 7 - 2 = 5$$

$\therefore f(x, y, z) is\ a\ homgeneous\ function\ of\ degree\ is$

$$n = 5$$

$By\ Euler\ theorem,\quad x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} + \text{z}\frac{\partial f}{\partial z} = \text{n}f$

$$x\frac{\partial e^w}{\partial x} + y\frac{\partial e^w}{\partial y} + \text{z}\frac{\partial e^w}{\partial z} = (5)e^w$$

$$x\frac{\partial w}{\partial x} + y\frac{\partial w}{\partial y} + \text{z}\frac{\partial w}{\partial z} = 5$$

7.Prove that $g(x, y) = xlog\ \left(\frac{y}{x}\right)$ is homogeneous, verify Euler's theorem for g

Soln

$$g(x, y) = xlog\ \left(\frac{y}{x}\right)$$

$$Degree = n = N.degree - D.degree$$

$$n = 2 - 1 = 1$$

$\therefore f(x, y) is\ a\ homgeneous\ function\ of\ degree\ is$

$$n = 1$$

$By\ Euler\ theorem, x\frac{\partial g}{\partial x} + y\frac{\partial g}{\partial y} = 1g$

LHS= $x\frac{\partial g}{\partial x} + y\frac{\partial g}{\partial y} = x\frac{\partial}{\partial x}\left(\text{x}\log\frac{y}{x}\right) + y\frac{\partial}{\partial y}\left(\text{x}\log\frac{y}{x}\right)$

$$= x\log\frac{y}{x} = g$$

Hence $x\frac{\partial g}{\partial x} + y\frac{\partial g}{\partial y} = 1g$

THEORY OF EQUATIONS

- $ax^3 + bx^2 + cx + d = 0$
- Sum of co-efficients = 0
  $\Rightarrow x = 1\ is\ a\ root$
- Sum of co-efficients $a + c = b + d$
  $\Rightarrow x = -1\ is\ a\ root$
- Otherwise try $x = 2\ or\ 3\ is\ a\ root.$

1.Solve $\boldsymbol{x^4 - 10x^3 + 26x^2 - 10x + 1 = 0}$

Soln:

Given $\boldsymbol{x^4 - 10x^3 + 26x^2 - 10x + 1 = 0}$

$$\boldsymbol{\left(x^2 + \frac{1}{x^2}\right) - 10\left(x + \frac{1}{x}\right) + 26 = 0}$$

$$\boldsymbol{y^2 - 2 - 10y + 26 = 0}$$

$$\boldsymbol{y^2 - 10y + 24 = 0}$$

$$\boldsymbol{(y - 6)(y - 4) = 0}$$

$$\boldsymbol{y = 6,\ \ y = 4}$$

Case(i) $x+\frac{1}{x}=6$	Case(ii) $x+\frac{1}{x}=4$
$\frac{x^2+1}{x}=6$	$\frac{x^2+1}{x}=4$
$x^2+1=6x$	$x^2+1=4x$
$x^2-6x+1=0$	$x^2-4x+1=0$
$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$	$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$
$=3\pm 2\sqrt{2}$	$=2\pm\sqrt{3}$

2.Solve $6x^4-35x^3+62x^2-35x+6=0$

Soln:

2	6	-35	62	-35	6
	0	12	-46	32	-6
3	6	-23	16	-3	0
	0	18	-15	3	
	6	-5	10	0	

Reduced equation is

$$6x^2-5x+1=0$$

$$\left(x-\frac{1}{3}\right)\left(x-\frac{1}{2}\right)=0$$

$$x=2,3,\frac{1}{2},\frac{1}{3}$$

3.Solve $6x^4-5x^3-38x^2-5x+6=0$ whose one of the roots is $\frac{1}{3}$ then find the other roots M-23

Soln:

$\frac{1}{3}$	6	-5	-38	-5	6
	0	2	-1	-13	-6
3	6	-3	-39	-18	0
	0	18	45	18	
	6	15	6	0	

Reduced equation is

$$6x^2+15x+6=0$$

$$\left(x+\frac{12}{6}\right)\left(x+\frac{3}{6}\right)=0$$

$$(x+2)\left(x+\frac{1}{2}\right)=0$$

$$x=\frac{1}{3},3,-2,-\frac{1}{2}$$

4.Solve $x^4+3x^3-3x-1=0$

Soln:

1	1	3	0	-3	-1
	0	1	4	4	1
-1	1	4	4	1	0
	0	-1	-3	-1	
	1	3	1	0	

Reduced equation is

$$x^2+3x+1=0$$

$$x=\frac{-3+\sqrt{5}}{2},\quad x=\frac{-3-\sqrt{5}}{2}$$

$$x=1,-1,\frac{-3+\sqrt{5}}{2},\frac{-3-\sqrt{5}}{2}$$

5.If $2+i$ and $3-\sqrt{2}$ are the roots of the equation $x^6-13x^5+62x^4-126x^3+65x^2+127x-140=0$ find all roots.

Soln: Given roots $2+i\ ,3-\sqrt{2}$

Other roots $2-i\ ,3+\sqrt{2}$

Let assume missing roots a and b.

S.R: $2+i\ +3-\sqrt{2}\ +2-i\ +3+\sqrt{2}+a+b=13$

$$10+a+b=13$$

$$a+b=3$$

P.R : $(2+i)(2-i)(3-\sqrt{2})(3+\sqrt{2})\,ab=-140$

$$5(7)ab=-140$$

$$ab=\frac{-140}{35}=-4$$

Reduced equation $x^2-3x-4=0$

$$(x-4)(x+1)=0$$

$$x=4,\quad x=-1$$

6.If $1+2i$ and $\sqrt{3}$ are the roots of the equation $x^6-3x^5-5x^4+22x^3-39x^2-39x+135=0$ find all roots.

Soln: Given roots $1+2i\ ,\sqrt{3}$

then other roots $1-2i\ ,-\sqrt{3}$

Let assume missing roots a and b.

SR $1+2i\ +\sqrt{3}\ +1-2i\ +(-\sqrt{3})+a+b=3$

$$a+b=1$$

PR $(1+2i)(1-2i)(\sqrt{3})(-\sqrt{3})\,ab=135$

$$5(-3)\,ab=135$$

$$ab=\frac{135}{-15}=-9$$

Reduced equation $x^2-x-9=0$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{1\pm\sqrt{1^2-4\times1\times-9}}{2\times1}=\frac{1\pm\sqrt{37}}{2}$$

7. Solve the eqn $3x^3-16x^2+23x-6=0$ if the product of two roots is 1.

Soln:

Let a,b,c are roots of $x^3-\frac{16}{3}x^2+\frac{23}{3}x-\frac{6}{3}=0$

$$abc=\quad c=2$$

2	3	-16	23	-6
	0	6	-20	6
	3	-10	3	0

Reduced equation

$$3x^2-10x+3=0$$

$$\left(x-\frac{9}{3}\right)\left(x-\frac{1}{3}\right)=0$$

$$x=2,3,\frac{1}{3}$$

8. Solve the equation $x^3 - 9x^2 + 14x + 24 = 0$ if it is given that two of its roots are in the ratio 3:2

Soln:

-1	1	-9	14	24
	0	-1	10	-24
	1	-10	24	0

Reduced equation

$x^2 - 10x + 24 = 0$

$(x-6)(x-4) = 0$

$x = -1, 4, 6$

Try yourself:

Solve $2x^3 + 11x^2 - 9x - 18 = 0$ J-23

9.Solve $2x^3 - 9x^2 + 10x = 3$ M-22

Soln: $2x^3 - 9x^2 + 10x - 3 = 0$

1	2	-9	10	-3
	0	2	-7	3
	2	-7	3	0

Reduced equation is

$2x^2 - 7x + 3 = 0$

$\left(x - \frac{6}{2}\right)\left(x - \frac{1}{2}\right) = 0$

$x = 1, 3, \frac{1}{2}$

10.Solve the equation $2x^3 - x^2 - 18x + 9 = 0$ if sum of two of its roots vanishes.

Soln: Let a, b, c be the roots of

$x^3 - \frac{1}{2}x^2 - \frac{18}{2}x + \frac{9}{2} = 0$

$\frac{1}{2}$	2	-1	-18	9
	0	1	0	-9
	2	0	-18	0

$a + b = 0$

$a + b + c = \frac{1}{2}$ c$= \frac{1}{2}$

Reduced equation is $2x^2 - 18 = 0$

$x = 3,\ x = -3$

$x = \frac{1}{2}, 3, -3$

11.Solve the equation $9x^3 - 36x^2 + 44x - 16 = 0$ if the roots form an A.P.

Soln:

Let $a - b, a, a + b$ be the roots of

$x^3 - \frac{36}{9}x^2 + \frac{44}{9}x - \frac{19}{9} = 0$ $a - b + a + a + b = 4$

$\frac{4}{3}$	9	-36	44	-16
	0	12	-32	16
	9	-24	12	0

$a = \frac{4}{3}$

Reduced equation is $9x^2 - 24x + 12 = 0$

$\left(x - \frac{18}{9}\right)\left(x - \frac{6}{9}\right) = 0$

$x = 2, \frac{2}{3}, \frac{4}{3}$

12.solve the equation $3x^3 - 26x^2 + 52x - 24 = 0$ if the roots form an G.P.

Soln:

Let $ar, a, \frac{a}{r}$ be the roots of $x^3 - \frac{26}{3}x^2 + \frac{52}{3}x - \frac{24}{3} = 0$

$a^3 = 8 \quad \Rightarrow \quad a = 2$

2	3	-26	52	-24
	0	6	-40	24
	3	-20	12	0

Reduced equation is

$3x^2 - 20x + 12 = 0$

$\left(x - \frac{18}{3}\right)\left(x - \frac{2}{3}\right) = 0$

$x = 2, 6, \frac{2}{3}$

13.Determine k and solve the equation $2x^3 - 6x^2 + 3x + k = 0$ if one root is twice the sum of the other two roots.

Soln:

2	2	-6	3	k
	0	4	-4	-2
	2	-2	-1	k-2

Let a, b, c be the roots of x^3

Given $a = 2(b + c)$

$a + b + c = 3$

$2a + 2b + 2c = 6$

$3a = 6$

$a = 2$

Reduced equation is

$2x^2 - 2x - 1 = 0$

$k - 2 = 0$

$k = 2$

$x = \frac{1 \pm \sqrt{3}}{2}$ $\quad x = 2, \frac{1+\sqrt{3}}{2}, \frac{1-\sqrt{3}}{2}$

14.Solve $(x-2)(x-7)(x-3)(x+2) + 19 = 0$

Soln: $(x-2)(x-3)(x-7)(x+2) + 19 = 0$

$(x^2 - 5x + 6)(x^2 - 5x - 14) + 19 = 0$

Put $x^2 - 5x = y$ $\quad (y+6)(y-14) + 19 = 0$

$y^2 - 8y - 84 + 19 = 0$

$y^2 - 8y - 65 = 0$

$y = 13, \quad y = -5$

Case(i) $y = 13$	Case(ii) $y = -5$
$x^2 - 5x = 13$	$x^2 - 5x = -5$
$x^2 - 5x - 13 = 0$	$x^2 - 5x + 5 = 0$
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{5 \pm \sqrt{77}}{2}$	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{5 \pm \sqrt{5}}{2}$

15.Solve $(2x-3)(6x-1)(3x-2)(x-2) - 5 = 0$

Soln: $(2x-3)(3x-2)(6x-1)(x-2) - 5 = 0$

$(6x^2 - 13x + 6)(6x^2 - 13x + 2) - 5 = 0$

Put $6x^2 - 13x = y$ $\quad (y+6)(y+2) - 5 = 0$

$y^2 + 8y + 7 = 0$

$y = -1, \quad y = -7$

Case(i) $y = -1$	Case(ii) $y = -7$
$6x^2 - 13x = -1$	$6x^2 - 13x = -7$
$6x^2 - 13x + 1 = 0$	$6x^2 - 13x + 7 = 0$
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	$x = 1, \frac{7}{6}$
$= \frac{13 \pm \sqrt{145}}{12}$	

16.Solve $(2x-1)(x+3)(x-2)(2x+3)+20=0$

Soln: $(2x-1)(2x+3)(x+3)(x-2)+20=0$

$(4x^2+4x-3)(x^2+x-6)+20=0$

put $x^2+x=y$ $\quad (4y-3)(y-6)+20=0$

$4y^2-27y+18+20=0$

$4y^2-27y+38=0$

$y=\frac{19}{4},\quad y=\frac{8}{4}=2$

Case(i)	Case(ii)
$y=\frac{19}{4}$	$y=2$
$x^2+x=\frac{19}{4}$	$x^2+x=2$
$4x^2+4x-19=0$	$x^2+x-2=0$
$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$	$x=-2, \quad x=1$
$=\frac{-1\pm2\sqrt{5}}{2}$	

17.Find the polynomial equation

(i) $2+i\sqrt{3}$ (ii) $2i+3$ (iii) $\sqrt{5}-\sqrt{3}$

(iv) $\sqrt{\frac{\sqrt{2}}{\sqrt{3}}}$ (v) $2-\sqrt{3}$ M-22

Soln:

(i) $x=2+i\sqrt{3}$ $x-2=i\sqrt{3}$ $(x-2)^2=(i\sqrt{3})^2$ $x^2-4x+4=-3$ $x^2-4x+7=0$	(ii) $x=2i+3$ $x-3=2i$ $(x-3)^2=(2i)^2$ $x^2-6x+9=-4$ $x^2-6x+13=0.$
(iii) $x=\sqrt{5}-\sqrt{3}$ $x^2=(\sqrt{5}-\sqrt{3})^2$ $x^2=5+3-2\sqrt{15}$ $x^2-8=-2\sqrt{15}$ $(x^2-8)^2=(-2\sqrt{15})^2$ $x^4-16x^2+64=60.$ $x^4-16x^2+4=0$	(iv) $x=\sqrt{\frac{\sqrt{2}}{\sqrt{3}}}$ $x^2=\frac{\sqrt{2}}{\sqrt{3}}$ $(x^2)^2=\left(\frac{\sqrt{2}}{\sqrt{3}}\right)^2$ $x^4=\frac{2}{3}$ $3x^4=2$ Or $3x^4-2=0$

18.Discuss the nature of the roots of equation

(i) $9x^9+2x^5-x^4-7x^2+2=0$

fun	Signs	No.of changes	No.of Real roots
$f(x)$	+ + - - +	2	2 +Ve
$f(-x)$	- - - - +	1	1 -Ve

No. of Imaginary roots = 9 - 3 = 6

(ii) $x^9+9x^7+7x^5+5x^3+3x=0$ J-23

$x(x^8+9x^6+7x^4+5x^2+3)=0$

$x=0$ is a root with multiplicity one

fun	Signs	No.of changes	No.of Real roots
$f(x)$	+ + + + +	0	0 +Ve
$f(-x)$	- - - - -	0	0 -Ve

No. of Imaginary roots = 9-1 = 8

(iii) $x^9-5x^8-14x^7=0 \Rightarrow x^7(x^2-5x-14)=0$

$x=0$ is a root with multiplicity seven

Fun	Signs	No.of changes	No.of Real roots
$f(x)$	+ - -	1	1 +Ve
$f(-x)$	- - +	1	1 -Ve

No. of Imaginary roots = 9 - 9 = 0

19. If the sides of a cubic box are increased by 1, 2, 3 units respectively to form a cuboid, then the volume is increased by 52 cubic units. Find the volume of the cuboid. S-21

Soln: $(x+1)(x+2)(x+3)-x^3=52$

$x^3+6x^2+11x+6-x^3=52$

$6x^2+11x+6=52$

$6x^2+11x-46=0$

$x=\frac{12}{6}, x=\frac{-23}{6}$

$x=2, \quad x=\frac{-23}{6}$ (*not possible*)

∴ The volume of the cuboid $(x+1)(x+2)(x+3)$

$=3\times4\times5=60$

20. Construct a cubic equation with roots

(i) 1, 2 , and 3

$(x-1)(x-2)(x-3)=0$

$x^3-6x^2+11x-6=0$

(ii) 1,1, and −2

$(x-1)(x-1)(x+2)=0$

$x^3-0x^2-3x+2=0$

(iii) 2 ,$\frac{1}{2}$, and 1.

$(x-2)\left(x-\frac{1}{2}\right)(x-1)=0$

$x^3-\frac{7}{2}x^2+\frac{7}{2}x-1=0$

$2x^3-7x^2+7x-2=0$

21. If α , β and γ are the roots of the cubic equation $x^3+2x^2+3x+4=0$, form a cubic equation whose roots are (i) 2α , 2β , 2γ (ii) $\frac{1}{\alpha},\frac{1}{\beta},\frac{1}{\gamma}$

(iii) $-\alpha,-\beta,-\gamma$ (iv) $\frac{\alpha}{2},\frac{\beta}{2},\frac{\gamma}{2}$

Soln:

(i) 2α , 2β , 2γ

$2^0x^3+2^12x^2+2^23x+2^34=0$

$x^3+4x^2+12x+32=0$

MATHS

(ii) $\frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\gamma}$

$4x^3 + 3x^2 + 2x + 1 = 0$

(iii) $-\alpha, -\beta, -\gamma$

$-x^3 + 2x^2 - 3x + 4 = 0$

$x^3 - 2x^2 + 3x - 4 = 0$

(iv) $\frac{\alpha}{2}, \frac{\beta}{2}, \frac{\gamma}{2}$

$$\left(\frac{1}{2}\right)^0 x^3 + \left(\frac{1}{2}\right)^1 2x^2 + \left(\frac{1}{2}\right)^2 3x + \left(\frac{1}{2}\right)^3 4 = 0$$

$$x^3 + x^2 + \frac{3}{4}x + \frac{4}{8} = 0$$

$$8x^3 + 8x^2 + 6x + 4 = 0$$

22. If α , β , γ and δ are the roots of polynomial eqn $2x^4 + 5x^3 - 7x^2 + 8 = 0$, find a quadratic equation whose roots are $\alpha+\beta+\gamma+\delta$ and $\alpha\beta\gamma\delta$.

Soln: Given $2x^4 + 5x^3 - 7x^2 + 8 = 0$

$$x^4 + \frac{5}{2}x^3 - \frac{7}{2}x^2 + 0x + \frac{8}{2} = 0$$

$\alpha+\beta+\gamma+\delta = \frac{-5}{2}$ $\qquad$ $\alpha\beta\gamma\delta = \frac{8}{2}$

Required equation is $x^2 - (S.R)x + (P.R) = 0$

$$x^2 - (\frac{-5}{2} + \frac{8}{2})x + (\frac{-5}{2} \times \frac{8}{2}) = 0$$

$$x^2 - \frac{3}{2}x - \frac{40}{4} = 0$$

$$2x^2 - 3x - 20 = 0$$

23. If the equations $x^2 + px + q = 0$ and $x^2 + p'x + q' = 0$ have a common root, show that it must be equal to $\frac{pq' - qp'}{q - q'}$ or $\frac{q - q'}{p - p'}$.

Soln: Let us assume 'a' be the common root

$a^2 + pa + q = 0$ $\qquad$ $\frac{a^2}{\begin{vmatrix} p & q \\ p' & q' \end{vmatrix}} = \frac{a}{\begin{vmatrix} q & 1 \\ q' & 1 \end{vmatrix}} = \frac{1}{\begin{vmatrix} 1 & p \\ 1 & p' \end{vmatrix}}$

$a^2 + p'a + q' = 0$ $\qquad$ $\frac{a^2}{pq' - qp'} = \frac{a}{q - q'} = \frac{1}{p' - p}$

$\frac{a^2}{pq' - qp'} = \frac{a}{q - q'}$ $\qquad$ $\frac{a}{q - q'} = \frac{1}{p' - p}$

$a = \frac{pq' - qp'}{q - q'}$ (or) $\qquad$ $a = \frac{q - q'}{p - p'}$

24.If α, β be the roots of $x^2 - 5x + 6 = 0$, find $\alpha^2 - \beta^2$

Soln:

$\alpha + \beta = 5, \quad \alpha\beta = 6$ $\qquad$ S-21

$$(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta = 5^2 - 4(6) = 1$$

$$\alpha - \beta = \pm 1$$

$$\alpha^2 - \beta^2 = (\alpha + \beta)(\alpha - \beta) = 5(\pm 1) = \pm 5$$

25.If α, β be the roots of $x^2 + 5x + 6 = 0$, find $\alpha^2 + \beta^2$

Soln: $\alpha + \beta = -5, \quad \alpha\beta = 6$ $\qquad$ J-22

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = (-5)^2 - 2(6) = 13$$

26.Solve

(i) $x^4 - 3x^2 - 4 = 0$

(ii) $x^4 - 14x^2 + 45 = 0$

Soln:

(i) $x^4 - 3x^2 - 4 = 0$ J-22	(ii) $x^4 - 14x^2 + 45 = 0$
$x^2 = -1$ $\quad$ $x^2 = 4$	$x^2 = 9$ $\quad$ $x^2 = 5$
$x = \pm\sqrt{-1}$ $\quad$ $x = \pm 2$	$x = \pm 3$ $\quad$ $x = \pm\sqrt{5}$
$= \pm i$	

27.Find the condition of the equation $x^3 + px^2 + qx + r = 0$ whose roots are *in A.P.* S-20

Soln: Let roots are $a - d, a, a + d$

sum of roots $a - d + a + a + d = -p$

$$3a = -p$$

$$a = \frac{-p}{3}$$

$$\left(\frac{-p}{3}\right)^3 + p\left(\frac{-p}{3}\right)^2 + q\left(\frac{-p}{3}\right) + r = 0$$

$$-p^3 + 3p^3 - 9pq + r = 0$$

$$2p^3 + r = 9pq$$

28.If α, β, and γ are the roots of the equation $x^3 + px^2 + qx + r = 0$ find the value of $\sum \frac{1}{\beta\gamma}$ in terms of the coefficients.

Soln:

$\alpha + \beta + \gamma = -p; \quad \alpha\beta\gamma = -r$

$$\sum \frac{1}{\beta\gamma} = \frac{\alpha + \beta + \gamma}{\alpha\beta\gamma} = \frac{-p}{-r} = \frac{p}{r}$$

29.If α and β are the roots of the quadratic equation $2x^2 - 7x + 13 = 0$ construct a quadratic equation whose roots are α^2 and β^2 . M-24

Soln:

put $x^2 = y$ $\qquad$ $2y - 7\sqrt{y} + 13 = 0$

$$2y + 13 = 7\sqrt{y}$$

$$2y + 13 = 7\sqrt{y}$$

$$(2y + 13)^2 = (7\sqrt{y})^2$$

$$4y^2 + 52y + 169 = 49y$$

$$4y^2 + 3y + 169 = 0$$

30.If α and β are the roots of the quadratic equation $17x^2 + 43x - 73 = 0$ construct a quadratic equation whose roots are α+2and β+2 .

Soln:

Roots increase by 2 then equation diminish by -2

	17	43	-73
-2	0	-34	-18
	17	9	-91
	0	-34	
	17	-25	

Required equation is

$17x^2 - 25x - 91 = 0$

31.Find the sum of squares of roots of $2x^4 - 8x^3 + 6x^2 - 3 = 0$

Soln: Let α, β, γ *and* δ be the roots of

$$x^4 - \frac{8}{2}x^3 + \frac{6}{2}x^2 - \frac{3}{2} = 0$$

$$\boldsymbol{\alpha+\beta+\gamma+\delta=4}$$

$$\boldsymbol{\alpha\beta+\alpha\gamma+\alpha\delta+\beta\gamma+\beta\delta+\gamma\delta=3}$$

$$\boldsymbol{\alpha^2+\beta^2+\gamma^2+\delta^2}$$

$$\boldsymbol{=(\alpha+\beta+\gamma+\delta)^2-2(\alpha\beta+\alpha\gamma+\alpha\delta+\beta\gamma+\beta\delta+\gamma\delta)}$$

$$\boldsymbol{=(4)^2-2(3)=10}$$

34 .Solve : $\boldsymbol{8x^{\frac{3}{2n}}-8x^{\frac{-3}{2n}}=63.}$

solu: $8x^{\frac{3}{2n}}-8x^{\frac{-3}{2n}}=63$

$$x^{\frac{3}{2n}}-x^{\frac{-3}{2n}}=\frac{64-1}{8}$$

$$x^{\frac{3}{2n}}-\frac{1}{x^{\frac{3}{2n}}}=8-\frac{1}{8}$$

$$x^{\frac{3}{2n}}=8, \qquad \frac{1}{x^{\frac{3}{2n}}}=\frac{1}{8}$$

$$x=(2^3)^{\frac{2n}{3}}=2^{2n}=4^n$$

34.Solve : $2\sqrt{\frac{x}{a}}+3\sqrt{\frac{a}{x}}=\frac{b}{a}+\frac{6a}{b}.$

solu: $2\sqrt{\frac{x}{a}}+3\sqrt{\frac{a}{x}}=\frac{b}{a}+\frac{6a}{b}$

$2\sqrt{\frac{x}{a}}=\frac{b}{a}$	$3\sqrt{\frac{a}{x}}=\frac{b}{a}$
$\sqrt{\frac{x}{a}}=\frac{b}{2a}$	$\sqrt{\frac{a}{x}}=\frac{b}{3a}$
$\frac{x}{a}=\frac{b^2}{4a^2}$	$\frac{a}{x}=\frac{b^2}{9a^2}$
$x=\frac{b^2}{4a}$	$x=\frac{9a^3}{b^2}$

35. Find all real numbers satisfying $4^x-3(2^{x+2})+2^5=0$.

solu: $4^x-3(2^{x+2})+2^5=0$

$$(2^2)^x-3(2^x\,2^2)+2^5=0$$

$$(2^x)^2-12(2^x)+32=0$$

$\boldsymbol{y=2^x}$ என்க

$$y^2-12y+32=0$$

$$\boldsymbol{(y-8)(y-4)=0}$$

$$\boldsymbol{y=8, y=4}$$

$$\boldsymbol{2^x=8=2^3, 2^x=4=2^2}$$

$$\boldsymbol{x=3, x=2}$$

PROBABILITY DISTRIBUTIONS

1. A six sided die is marked '1' on one face, '2' on two of its faces, and '3' on remaining three faces. The die is rolled twice. If X denotesthe total score in two throws. Find (i) The probability mass function. (ii) The cumulative distribution function.(iii) $P(3\le X<6)$ (iv) $P(X\ge 4)$.

Soln: The random variable X takes the value 2,3,4,5 and 6.

+	1	2	2	3	3	3
1	2	3	3	4	4	4
2	3	4	4	5	5	5
2	3	4	4	5	5	5
3	4	5	5	6	6	6
3	4	5	5	6	6	6
3	4	5	5	6	6	6

X	2	3	4	5	6
PMF	$\frac{1}{36}$	$\frac{4}{36}$	$\frac{10}{36}$	$\frac{12}{36}$	$\frac{9}{36}$
CDF	$\frac{1}{36}$	$\frac{5}{36}$	$\frac{15}{36}$	$\frac{27}{36}$	$\frac{36}{36}$

iii)$P(3\le X<6)=P(x=3)+P(x=4)+P(x=5)$

$$=\frac{4}{36}+\frac{10}{36}+\frac{12}{36}=\frac{26}{36}$$

(iv) $P(X\ge 4)=P(x=4)+P(x=5)+P(x=6)$

$$=\frac{10}{36}+\frac{12}{36}+\frac{9}{36}=\frac{31}{36}$$

2. A six sided die is marked '1' on one face, '3' on two of its faces ,and '5' on remaining three faces. The die is rolled twice. If X denotes the total score in two throws. Find (i) The probability mass function (ii) The cumulative distribution function.(iii) $P(4\le X<10)$ (iv) $P(X\ge 6)$.

Soln:

The random variable X takes the value 2,4,6,8 and 10.

+	1	3	3	5	5	5
1	2	4	4	6	6	6
3	4	6	6	8	8	8
3	4	6	6	8	8	8
5	6	8	8	10	10	10
5	6	8	8	10	10	10
5	6	8	8	10	10	10

X	2	4	6	8	10
PMF	$\frac{1}{36}$	$\frac{4}{36}$	$\frac{10}{36}$	$\frac{12}{36}$	$\frac{9}{36}$
CDF	$\frac{1}{36}$	$\frac{5}{36}$	$\frac{15}{36}$	$\frac{27}{36}$	$\frac{36}{36}$

iii) $P(4\le X<10)=P(x=4)+P(x=6)+P(x=8)$

$$=\frac{4}{36}+\frac{10}{36}+\frac{12}{36}=\frac{26}{36}$$

(iv) $P(X\ge 6)=P(x=6)+P(x=8)+P(x=10)$

$$=\frac{10}{36}+\frac{12}{36}+\frac{9}{36}=\frac{31}{36}$$

3.A random variable X has the following probability mass function.

x	1	2	3	4	5	6
$f(x)$	k	$2k$	$6k$	$5k$	$6k$	$10k$

Find (i) $P(2<X<6)$ (ii) $P(2\le X<5)$ (iii) $P(X\le 4)$ (iv) $P(3<X)$

Soln: Given f is P.M.F

$$\therefore \sum f(x)=1$$

MATHS

$k + 2k + 6k + 5k + 6k + 10k = 1$

$30k = 1 \implies k = \frac{1}{30}$

x	1	2	3	4	5	6
$f(x)$	$\frac{1}{30}$	$\frac{2}{30}$	$\frac{6}{30}$	$\frac{5}{30}$	$\frac{6}{30}$	$\frac{10}{30}$

ii) $P(2 < X < 6) = P(x = 3) + P(x = 4) + P(x = 5)$

$= \frac{6}{30} + \frac{5}{30} + \frac{6}{30} = \frac{17}{30}$

(ii) $P(2 \le X < 5) = P(x = 2) + P(x = 3) + P(x = 4)$

$= \frac{2}{30} + \frac{6}{30} + \frac{5}{30} = \frac{13}{30}$

iii) $P(X \le 4) = P(x = 1) + P(x = 2) + P(x = 3) + P(x = 4)$

$= \frac{1}{30} + \frac{2}{30} + \frac{6}{30} + \frac{5}{30} = \frac{14}{30}$

(iv) $P(3 < X) = P(x = 4) + P(x = 5) + P(x = 6)$

$= \frac{5}{30} + \frac{6}{30} + \frac{10}{30} = \frac{21}{30}$

4. A random variable X has the following probability mass function.

x	1	2	3	4	5
$f(x)$	k^2	$2k^2$	$3k^2$	$2k$	$3k$

Find (i) $P(2 \le X < 5)$ (ii) $P(3 < X)$

Soln: Given f is P.M.F $\therefore \sum f(x) = 1$

$k^2 + 2k^2 + 3k^2 + 2k + 3k = 1$

$6k^2 + 5k - 1 = 0$

$k = -1$, $k = \frac{1}{6}$

x	1	2	3	4	5
$f(x)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{2}{6} = \frac{12}{36}$	$\frac{3}{6} = \frac{18}{36}$

i) $P(2 \le X < 5) = P(x = 2) + P(x = 3) + P(x = 4)$

$= \frac{2}{36} + \frac{3}{36} + \frac{12}{36} = \frac{17}{36}$

(ii) $P(3 < X) = P(x = 4) + P(x = 5) = \frac{12}{36} + \frac{18}{36} = \frac{30}{36}$

5.The cumulative distribution function of a discrete random variable is given by find

$$F(x) = \begin{cases} 0 & ; -\infty < x < 0 \\ \frac{1}{2} & ; 0 \le x < 1 \\ \frac{3}{5} & ; 1 \le x < 2 \\ \frac{4}{5} & ; 2 \le x < 3 \\ \frac{9}{10} & ; 3 \le x < 4 \\ 1 & ; 4 \le x < \infty \end{cases}.$$

(i) The Probability mass function $f(x)$

(ii) $P(X < 3)$

(iii) (iii) $P(X \ge 2)$

Soln.

i)The values of the discrete random variable X are 0,1,2,3,4.

x	0	1	2	3	4
$f(x)$	$\frac{1}{2} = \frac{5}{10}$	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{1}{10}$	$\frac{1}{10}$

ii) $P(X < 3) = P(X = 0) + P(X = 1) + P(X = 2)$

$= \frac{5}{10} + \frac{1}{10} + \frac{2}{10} = \frac{8}{10} = \frac{4}{5}$

iii) $P(X \ge 2) = P(X = 2) + P(X = 3) + P(X = 4)$

$= \frac{2}{10} + \frac{1}{10} + \frac{1}{10} = \frac{4}{10} = \frac{2}{5}$

6.The cumulative distribution function of a discrete random variable is given by

$$F(x) = \begin{cases} 0 & ; -\infty < x < -1 \\ 0.15 & ; -1 \le x < 0 \\ 0.35 & ; 0 \le x < 1 \\ 0.60 & ; 1 \le x < 2 \\ 0.85 & ; 2 \le x < 3 \\ 1 & ; 3 \le x < \infty \end{cases}$$

Find (i)the probability mass function

(ii) $p(X < 1)$ and(iii) $P(X \ge 2)$

Soln:

The values of the discrete random variable X are -1,0,1,2,3.

(i)The Probability mass function $f(x)$:

x	-1	0	1	2	3
$F(x)$	0.15	0.35	0.60	0.85	1
$f(x)$	0.15	0.20	0.25	0.25	0.15

ii) $P(X < 1) = P(X = -1) + P(X = 0)$

$= 0.15 + 0.20 = 0.35$

iii) $P(X \ge 2) = P(X = 2) + P(X = 3)$

$= 0.25 + 0.15 = 0.40$

4.INVERSE TRIGONOMETRY

5 Marks

Important Hints:

$tan^{-1}x + tan^{-1}y = tan^{-1}\left(\frac{x+y}{1-xy}\right)$,

$tan^{-1}x - tan^{-1}y = tan^{-1}\left(\frac{x-y}{1+xy}\right)$

$cos^{-1}x + cos^{-1}y = cos^{-1}(xy - \sqrt{(1-x^2)}\ \sqrt{1-y^2}\ )$

1.If $a_1, a_2, a_3 \dots a_n$ is an arithmetic progression with common difference d, prove that

$$\tan\left[tan^{-1}\left(\frac{d}{1+a_1a_2}\right) + tan^{-1}\left(\frac{d}{1+a_2a_3}\right) + \cdots + tan^{-1}\left(\frac{d}{1+a_na_{n-1}}\right)\right] = \frac{a_n - a_1}{1+a_1a_n}$$

soln

$tan^{-1}\left(\frac{d}{1+a_1a_2}\right) = tan^{-1}\left(\frac{a_2-a_1}{1+a_1a_2}\right) = tan^{-1}a_2 - tan^{-1}a_1$

$tan^{-1}\left(\frac{d}{1+a_2a_3}\right) = tan^{-1}\left(\frac{a_3-a_2}{1+a_2a_3}\right) = tan^{-1}a_3 - tan^{-1}a_2$

$tan^{-1}\left(\frac{d}{1+a_na_{n-1}}\right) = tan^{-1}\left(\frac{a_n-a_{n-1}}{1+a_{n-1}a_n}\right)$

$= tan^{-1}a_n - tan^{-1}a_{n-1}$

$tan^{-1}\left(\frac{d}{1+a_1a_2}\right) + tan^{-1}\left(\frac{d}{1+a_2a_3}\right) + .. + tan^{-1}\left(\frac{d}{1+a_na_{n-1}}\right)$

$= tan^{-1}a_n - tan^{-1}a_1$

$tan\left[tan^{-1}\left(\frac{d}{1+a_1a_2}\right) + tan^{-1}\left(\frac{d}{1+a_2a_3}\right) + \cdots + tan^{-1}\left(\frac{d}{1+a_na_{n-1}}\right)\right]$

$= tan[\ tan^{-1}a_n - tan^{-1}a_1]$

$= \tan\left[tan^{-1}\left(\frac{a_n-a_1}{1+a_1a_n}\right)\right] = \frac{a_n-a_1}{1+a_1a_n}$

2.prove that: $tan^{-1}x + tan^{-1}\left(\frac{2x}{1-x^2}\right) = tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$, $|x| < 1/\sqrt{3}$

soln

$tan^{-1}x + tan^{-1}\left(\frac{2x}{1-x^2}\right) = tan^{-1}\left(\frac{x + \frac{2x}{1-x^2}}{1 - x\left(\frac{2x}{1-x^2}\right)}\right)$

$= tan^{-1}\left(\frac{x - x^3 + 2x}{1 - x^2 - 2x^2}\right)$

$= tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right)$

3. Show that $tan^{-1}x + tan^{-1}y + tan^{-1}z = tan^{-1}\left(\frac{x+y+z-xyz}{1-xy-yz-zx}\right)$

Soln:

$tan^{-1}x + tan^{-1}y + tan^{-1}z$

$= tan^{-1}\left(\frac{x+y}{1-xy}\right) + tan^{-1}(z)$

$= tan^{-1}\left(\frac{\left(\frac{x+y}{1-xy}\right) + z}{1 - \left(\frac{x+y}{1-xy}\right)z}\right)$

$= tan^{-1}\left(\frac{[x+y+z(1-xy)]/(1-xy)}{[1-xy-(xz+yz)]/(1-xy)}\right)$

$= tan^{-1}\left(\frac{x+y+z-xyz}{1-xy-yz-zx}\right)$

4. If $tan^{-1}x + tan^{-1}y + tan^{-1}z = \pi$ then show that $x+y+z = xyz$

Soln:

$tan^{-1}x + tan^{-1}y + tan^{-1}z$

$= tan^{-1}\left(\frac{x+y}{1-xy}\right) + tan^{-1}(z)$

$= tan^{-1}\left(\frac{\left(\frac{x+y}{1-xy}\right) + z}{1 - \left(\frac{x+y}{1-xy}\right)z}\right)$

$= tan^{-1}\left(\frac{[x+y+z(1-xy)]/(1-xy)}{[1-xy-(xz+yz)]/(1-xy)}\right)$

$tan^{-1}x + tan^{-1}y + tan^{-1}z$

$= tan^{-1}\left(\frac{x+y+z-xyz}{1-xy-yz-zx}\right) = \pi$

$\frac{x+y+z-xyz}{1-xy-yz-zx} = tan\pi = 0$

$x+y+z-xyz = 0$

$x+y+z = xyz$

5. Find the number of solutions of the equation $tan^{-1}(x-1) + tan^{-1}(x) + tan^{-1}(x+1) = tan^{-1}(3x)$

Soln:

$tan^{-1}(x-1) + tan^{-1}(x+1) = tan^{-1}(3x) - tan^{-1}(x)$

$tan^{-1}\left(\frac{(x-1)+(x+1)}{1-(x-1)(x+1)}\right) = tan^{-1}\left(\frac{3x-x}{1+3x(x)}\right)$

$\frac{2x}{1-(x^2-1)} = \frac{2x}{1+3x^2}$

$2x(1+3x^2) = 2x(2-x^2)$

$2x + 6x^3 = -2x^3 + 4x$

$8x^3 - 2x = 0$

∴ Given equation has 3 solutions

6. Solve $\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$.

Soln: Given $\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$

$\tan^{-1}\left(\frac{\frac{x-1}{x-2} + \frac{x+1}{x+2}}{1 - \left(\frac{x-1}{x-2}\right)\left(\frac{x+1}{x+2}\right)}\right) = \frac{\pi}{4}$

$\frac{(x-1)(x+2) + (x+1)(x-2)}{(x-2)(x+2) - (x-1)(x+1)} = \tan\frac{\pi}{4}$

$\frac{x^2-x+2x-2+x^2+x-2x-2}{x^2-4-(x^2-1)} = 1$

MATHS

$$\frac{2x^2-4}{x^2-4-x^2+1} = 1$$

$$\frac{2x^2-4}{-3} = 1$$

$$\Rightarrow 2x^2 = -3+4$$

$$2x^2 = 1 \Rightarrow x^2 = \frac{1}{2}$$

$$x = \pm\frac{1}{\sqrt{2}}.$$

7.If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$ and $0 < x, y, z < 1$, then show that $x^2 + y^2 + z^2 + 2xyz = 1$.

Soln

Given, $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$

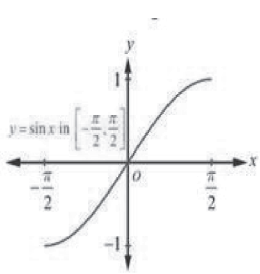
$$\cos^{-1}x + \cos^{-1}y = \pi - \cos^{-1}z$$

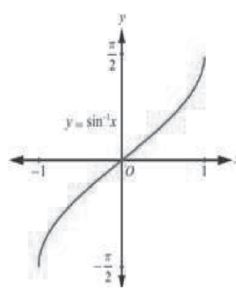
$$\cos^{-1}(xy - \sqrt{1-x^2}\sqrt{1-y^2}) = \cos^{-1}(-z)$$

$$-z = xy - \sqrt{1-x^2}\sqrt{1-y^2}$$

$$x^2 + y^2 + z^2 + 2xyz = 1.$$

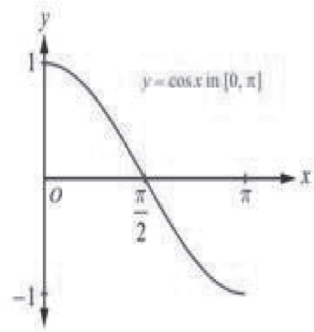
8.Draw the curve $\sin x$ in the domain $[-\frac{\pi}{2}, \frac{\pi}{2}]$ and $\sin^{-1}x$ in [-1,1]

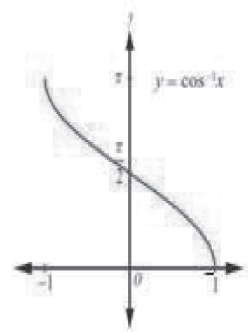




Domain :$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \to [-1, 1]$ Domain: $[-1, 1] \to \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

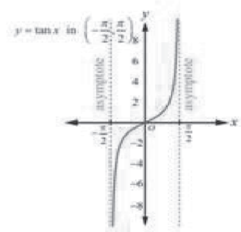
9.Draw the curve $\cos x$ in the domain $[0.\pi]$ and $\cos^{-1}x$ in $[-1, 1]$

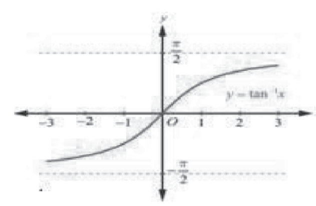




Domain:$[0, \pi] \to [-1, 1]$ Domain :$[-1, 1] \to [0, \pi]$

10.Draw the curve $\tan x$ in the domain $(-\frac{\pi}{2}, \frac{\pi}{2})$ and $\tan^{-1}x$ in R





Domain: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \to R$ Domain: $R \to \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

DIFFERENTIAL EQUATIONS

1. The rate of increase in the number of bacteria in a certain bacteria culture is proportional to the number present. Given that the number triples in 5 hours, find how many bacteria will be present after 10 hours?

Soln:

Let A be the no.of bacteria at present

$$\frac{dA}{dt} \propto A$$

t	A
0	A_0
5	$3A_0$
10	?

$\Rightarrow \frac{dA}{dt} = kA$

$\Rightarrow \quad A = Ce^{kt} ----\rightarrow (1)$

Let A_0 be the initial no. of bacteria

i.e, $t = 0, A = A_0$

$(1) \Rightarrow A_0 = Ce^0 \Rightarrow A_0 = C$

$\therefore A = A_0 e^{kt} ----\rightarrow (2)$

Put $A = 3A_0$ at $t = 5$

$(2) \Rightarrow \quad 3A_0 = A_0 e^{5k} \quad \Rightarrow \quad 3 = e^{5k}$

A=? at $t = 10 \quad A = A_0 e^{10} \quad \Rightarrow A = A_0(e^{5k})^2$

$\Rightarrow A = A_0(3)^2 = 9A_0$

....

2. Find the population of a city at any time *t*, given that the rate of increase of population is proportional to the population at that instant and that in a period of 40 years the population increased from 3,00,000 to 4,00,000.

Soln:

Let A be the population of a city at present

$$\frac{dA}{dt} \propto A$$

t	A
0	300000
40	400000
t	?

$\Rightarrow \frac{dA}{dt} = kA$

$\Rightarrow \quad A = Ce^{kt} ----\rightarrow (1)$

Let 3,00,000 be the initial population of a city

i.e, $t = 0, A = 30000$

$(1) \Rightarrow 3{,}00{,}000 = Ce^0 \Rightarrow 3{,}00{,}000 = C$

$\therefore A = 3{,}00{,}000e^{kt} ----\rightarrow (2)$

Put $A = 4{,}00{,}000$ at $t = 40$

$(2) \Rightarrow \quad 4{,}00{,}000 = 3{,}00{,}000e^{40k}$

$\Rightarrow \frac{4}{3} = e^{40} \quad \Rightarrow log\left(\frac{4}{3}\right) = 40k \quad \Rightarrow \quad k = \frac{1}{40} log\left(\frac{4}{3}\right)$

$A = 3{,}00{,}000e^{\frac{1}{40}log\left(\frac{4}{3}\right)t} \quad \Rightarrow A = 3{,}00{,}000\left(\frac{4}{3}\right)^{\frac{t}{40}}$

....

3.The engine of a motor boat moving at 10 *m* / *s* is shut off. Given that the retardation at any subsequent time (after shutting off the engine) equal to the velocity at that time. Find the velocity after 2 seconds of switching off the engine.

Soln:

Let A be the no.of bacteria at present

$$\frac{dA}{dt} \propto A$$

t	A
0	10
2	?

$\Rightarrow \frac{dA}{dt} = -A$

$\Rightarrow \quad A = Ce^{-t} ----\rightarrow (1)$

Let 10 be the initial velocity

i.e, $t = 0, A = 10$

$(1) \Rightarrow 10 = Ce^0 \Rightarrow 10 = C$

$\therefore A = 10e^{-t} ----\rightarrow (2)$

A=? at $t = 2$

$(2) \Rightarrow A = 10e^{-2}\ (or) \quad \Rightarrow A = 10/_{e^2}$

....

4.Suppose a person deposits 10,000 Indian rupees in a bank account at the rate of 5% per annum compounded continuously. How much money will be in his bank account 18 months later?

Soln:

Let A be the no.of bacteria at present

$$\frac{dA}{dt} \propto A$$

K=0.05

t	x
0	10,000
1.5	?

$\Rightarrow \frac{dA}{dt} = 0.05A$

$\Rightarrow \quad A = Ce^{0.05} ----\rightarrow (1)$

Let A=10,000 be the initial deposit

i.e, $t = 0, A = 10000$

$(1) \Rightarrow 10{,}000 = Ce^0 \Rightarrow 10{,}000 = C$

$\therefore A = 10{,}000e^{0.05t} ----\rightarrow (2)$

put $t = \frac{3}{2}$ =1.5 then A=?

$(2) \Rightarrow A = 10{,}000e^{0.05\left(\frac{3}{2}\right)} \quad \Rightarrow A = 10{,}000e^{0.075}$

....

5.Assume that the rate at which radioactive nuclei decay is proportional to the number of such nuclei that are present in a given sample. In a certain sample 10% of the original number of radioactive nuclei have undergone disintegration in a period of 100 years. What percentage of the original radioactive nuclei will remain after 1000 years?

Soln:

Let A be the no.of bacteria at present

$$\frac{dA}{dt} \propto A$$

$\Rightarrow \frac{dA}{dt} = -kA$

$\Rightarrow \quad A = Ce^{-k} ----\rightarrow (1)$

Let A_0 be the initial no. of bacteria

t	A
0	100
100	90
1000	?

i.e, $t = 0, A = A_0$

$(1) \Rightarrow A_0 = Ce^0 \Rightarrow A_0 = C$

$\therefore A = A_0 e^{-kt} ----\rightarrow (2)$

Put $A = \frac{9}{10}A_0$ at $t = 100$

$(2) \Rightarrow \quad \frac{9}{10}A_0 = A_0 e^{-100k}$

$\Rightarrow \quad \frac{9}{10} = e^{-100k} \Rightarrow k = \frac{-1}{100} log\left(\frac{9}{10}\right)$

$t = 1000$, then A=?

$(2) \Rightarrow A = A_0 e^{-1000k} \Rightarrow A = A_0 e^{-1000\left[\frac{-1}{100}log\left(\frac{9}{10}\right)\right]}$

$\Rightarrow \quad \frac{A}{A_0} = e^{10log\left(\frac{9}{10}\right)} = \left(\frac{9}{10}\right)^{10}$

Percentage of radioactive nuclei

$\frac{A}{A_0} \times 100 = \left(\frac{9}{10}\right)^{10} \times 100\ (or)\ \frac{9^{10}}{10^8}\%$

....

6.Water at temperature 100°C cools in 10 minutes to 80°C in a room temperature of 25°C .Find(i) The temperature of water after 20 minutes (ii) The time when the temperature is 40°C.
$\left[log\frac{11}{15} = -0.3101; log5 = 1.6094\right]$.

Soln:

$$\frac{dT}{dt} \propto T - T_m$$

t	T	s
0	100	25
10	80	
20	?	
?	40	

$\Rightarrow \frac{dT}{dt} = k(T - T_m)$

$\Rightarrow T - T_m = Ce^{kt}$

$\Rightarrow T - 25 = Ce^{kt} - - - \longrightarrow (1)$

Put T=100 at t=0

$(1) \Rightarrow C = 75$,

$(1) \Rightarrow T - 25 = 75e^{kt} - - - \longrightarrow (2)$

Put T=80 at t=10,

(2) $\Rightarrow$ 80-25 =$75e^{10k}$ $\Rightarrow 55 = 75e^{10}$

$\frac{55}{75} = e^{10} \Rightarrow e^{10k} = \frac{11}{15} \Rightarrow k = \frac{1}{10}log\left(\frac{11}{15}\right)$

Put t=20, T=?

$(2) \Rightarrow T - 25 = 75e^{20k}$ $\Rightarrow$T=25+75$(e^{10k})^2$ $\Rightarrow$T=25+75$\left(\frac{11}{15}\right)^2$

$\Rightarrow T = 65.33$

Put T=40, t=?

$(2) \Rightarrow 40 - 25 = 75e^{kt} \Rightarrow \frac{15}{75} = e^{kt} \Rightarrow log\left(\frac{15}{75}\right) = kt$

$$\Rightarrow log\left(\frac{15}{75}\right)$$
$$= \frac{1}{10}log\left(\frac{11}{15}\right)t$$

$t = \frac{10log\left(\frac{15}{75}\right)}{log\left(\frac{11}{15}\right)} \Rightarrow t = 53.46\ min$

....

7. At 10.00 A.M. a woman took a cup of hot instant coffee from her microwave oven and placed it on a nearby Kitchen counter to cool. At this instant the temperature of the coffee was 180°F, and 10 minutes later it was 160°F . Assume that constant temperature of the kitchen was 70°F . (i) What was the temperature of the coffee at 10.15A.M.? (ii) The woman likes to drink coffee when its temperature is between 130°F and 140°F.between what times should she have drunk the coffee?

Soln:

$\frac{dT}{dt} \propto T - T_m - - - \longrightarrow (1)$

t	T	s
0	180	70
10	160	
15	?	
?	130	
?	140	

$\Rightarrow \frac{dT}{dt} = k(T - T_m)$

$\Rightarrow T - T_m = Ce^{kt}$

$\Rightarrow T - 70 = Ce^{kt} - - - \longrightarrow (2)$

At T=180 at t=0

$(2) \Rightarrow C = 110$

$(2) \Rightarrow T - 70 = 110e^{kt} - - - \longrightarrow (3)$

At T=1600 at t=10

$(3) \Rightarrow 160 - 70 = 110e^{10k}$

$\Rightarrow e^{10} = \frac{90}{110}$

$\Rightarrow e^k = \left(\frac{9}{11}\right)^{\frac{1}{10}}$

At t=15 , T=?

$(3) \Rightarrow T - 70 = 110e^{kt}$

$\Rightarrow$T=70+110e^{15k}

$\Rightarrow$T=70+110$\left(\frac{9}{11}\right)^{\frac{15}{10}}$

$\Rightarrow T = 151.33$

At T=130, t=?

$(3) \Rightarrow 130 - 70 = 110e^{kt}$

$\Rightarrow \frac{60}{110} = e^{kt}$

$\Rightarrow log\left(\frac{6}{11}\right) = kt$

$\Rightarrow log\left(\frac{6}{11}\right) = \frac{1}{10}log\left(\frac{9}{11}\right)t$

$t = \frac{10log\left(\frac{6}{11}\right)}{log\left(\frac{9}{11}\right)} \Rightarrow t = 30.20\ min$

At T=140, t=?

$140 - 70 = 110e^{kt} \Rightarrow \frac{70}{110} = e^{kt} \Rightarrow log\left(\frac{7}{11}\right) = kt \Rightarrow$

$log\left(\frac{7}{11}\right) = \frac{1}{10}log\left(\frac{9}{11}\right)t$

$t = \frac{10l\ \ \frac{7}{11})}{log\left(\frac{9}{11}\right)} \Rightarrow t = 22.52\ min$

She drunk coffee between 10.22 min to 10.30 min.

....

8.A pot of boiling water at 100°C is removed from a stove at time $t = 0$ and left to cool in the kitchen. After 5 minutes, the water temperature has decreased to 80°C , and another 5 minutes later it has dropped to 65°C. Determine the temperature of the kitchen.

Soln:

t	T	s
0	100	?
5	80	
10	65	

$\frac{dT}{dt} \propto T - T_m$

$\Rightarrow \frac{dT}{dt} = k(T - T_m)$

$\Rightarrow T - T_m = Ce^{kt} - - - \longrightarrow (1)$

At T=100 at t=0

$(1) \Rightarrow 100 - T_m = C$

$(1) \Rightarrow T - T_m = (100 - T_m)e^{kt} - - - \longrightarrow (2)$

At T=80 at t=5

$(2) \Rightarrow 80 - T_m = (100 - T_m)e^{5k}$

$\Rightarrow e^{5k} = \frac{80-T_m}{100-T_m}$

At T=65 at, t=10

$(2) \Rightarrow 65 - T_m = (100 - T_m)e^{10k}$

$\Rightarrow e^{10} = \frac{65 - T_m}{100 - T_m}$

$\frac{65-T_m}{100-T_m} = (e^{5k})^2 = \left(\frac{80-T_m}{100-T_m}\right)^2$

$(65 - T_m)(100 - T_m)$=$(80 - T_m)^2$

$T_m = 20$

9.A tank initially contains 50 litres of pure water. Starting at time $t = 0$ a brine containing with 2 grams of dissolved salt per litre flows into the tank at the rate of 3 litres per minute. The mixture is kept uniform by stirring and the well-stirred mixture simultaneously flows out of the tank at the same rate. Find the amount of salt present in the tank at any time $t > 0$.

Soln: $\frac{dA}{dt} = IN - OUT$

$\frac{dA}{dt} = 50 - 0.01A$

$\frac{dA}{dt} = -0.01(A - 50000)$

$\Rightarrow \quad A - 5000 = Ce^{-0.01t} \longrightarrow (1)$

t	A
0	100

Put A = 100, t = 0

$(1) \Rightarrow \quad 100 - 5000 = C \quad \Rightarrow C = -4900$

$\therefore A - 5000 = -4900e^{-0.01}$

10.The growth of a population is proportional to the number present. If the population of a colony doubles in 50 years, in how many years will the population become triple.

Soln: Let A be the no.of bacteria at present

$\frac{dA}{dt} \propto A$

$\Rightarrow \frac{dA}{dt} = kA$

t	T	S
0	70	50
2	60	
t_1	98.6	

$\Rightarrow \quad A = Ce^{kt} \longrightarrow (1)$

Let A_0 be the initial no. of bacteria

i.e, $t = 0$,$A = A_0$

$(1) \Rightarrow A_0 = Ce^0 \Rightarrow A_0 = C$

$\therefore A = A_0e^{kt} \longrightarrow (2)$

Put $A = 2A_0$ at $t = 50$

$(2) \Rightarrow 2A_0 = A_0e^{50} \Rightarrow 2 = e^{50}$

$\Rightarrow k = \frac{1}{50}log2$

Put A=$3A_0$ at t =?

$(2) \Rightarrow 3A_0 = A_0e^{kt}$

$\Rightarrow 3 = e^{kt} \Rightarrow log3 = kt$

$\Rightarrow log3 = (\frac{1}{50}log2)t$

$t = 50\frac{log3}{log2}$.

t	A
0	A_0
50	$2A_0$
?	$3A_0$

11. A radioactive isotope has an initial mass 200mg, which two years later is 50mg . Find the expression for the amount of the isotope remaining at any time. What is its half-life? (half-life means the time taken for the radioactivity of a specified isotope to fall to half its original value).

Soln: Let A mass at present

$\frac{dA}{dt} \propto A$

t	A
0	200
2	150
?	100

$\Rightarrow \frac{dA}{dt} = -kA$

$\Rightarrow \quad A = Ce^{-kt} \longrightarrow (1)$

Let 200 be the initial mass

i.e, $t = 0, A = 200$

$(1) \Rightarrow 200 = Ce^0 \Rightarrow 200 = C$

$\therefore A = 200e^{-k} \longrightarrow (2)$

Put $A = 150$ at $t = 2$

$(2) \Rightarrow 150 = 200e^{-2k}$

$\Rightarrow \frac{3}{4} = e^{-2k} \Rightarrow log\left(\frac{3}{4}\right) = -2k$

Put A=100 at t =?

$(2) \Rightarrow 100 = 200e^{kt}$

$\Rightarrow \frac{1}{2} = e^{kt} \quad \Rightarrow log\left(\frac{1}{2}\right) = kt$

From 1,2 $\frac{-kt}{-2k} = \frac{log\left(\frac{1}{2}\right)}{log\left(\frac{3}{4}\right)}$

$\Rightarrow t = 2\,\frac{log\left(\frac{1}{2}\right)}{log\left(\frac{3}{4}\right)}$

12. In a murder investigation, a corpse was found by a detective at exactly 8 p.m. Being alert, the detective also measured the body temperature and found it to be 70°F. Two hours later, the detective measured the body temperature again and found it to be 60°F. If the room temperature is 50°F, and assuming that the body temperature of the person before death was 98.6°F, at what time did the murder occur? [$\log(2.43) = 0.88789; \log(0.5) = -0.69315$].

Soln

$\frac{dT}{dt} \propto T - T_m$

$\Rightarrow \frac{dT}{dt} = k(T - T_m)$

$\Rightarrow T - T_m = Ce^{kt}$

$\Rightarrow T - 50 = Ce^{kt} \longrightarrow (1)$

Put T=70 at t=0

$(1) \Rightarrow C = 20$

$(1) \Rightarrow \quad T - 50 = 20e^{kt} \longrightarrow (2)$

Put T=60 at t=2

$(2) \Rightarrow$ 60-50 =$20e^{2k}$

$\Rightarrow \quad e^{2k} = \frac{1}{2} \quad \Rightarrow k = \frac{1}{2}log\left(\frac{1}{2}\right)$

Put T=98.6, t=?

$(2) \Rightarrow 98.6 - 50 = 20e^{kt}$

$\Rightarrow \frac{48.6}{20} = e^{kt} \Rightarrow log(2.43) = kt \Rightarrow log(2.43) = \frac{1}{2}log\left(\frac{1}{2}\right)t$

$t = \frac{2log(2.43)}{log\left(\frac{1}{2}\right)} \quad \Rightarrow t \cong -2.56$

$\therefore$ the murder time is $8 - 2.56 \cong 5{:}30\ PM$

13. A tank contains 1000 litres of water in which 100 grams of salt is dissolved. Brine (Brine is a high-concentration solution of salt usually sodium chloride) in water runs in a rate of 10 litres per minute, and each litre contains 5grams of dissolved salt. The mixture of the tank is kept uniform by stirring. Brine runs out at 10 litres per minute. Find the amount of salt at any time *t* .

Soln: $\frac{dA}{dt} = IN - OUT$

$\frac{dA}{dt} = 6 - \frac{3}{50}A = \frac{300-3A}{50}$

t	A
0	0

$\frac{dA}{dt} = \frac{-3}{50}(A - 100)$

$\Rightarrow \quad A - 100 = Ce^{\frac{-3}{50}t} \longrightarrow (1)$

Put A = 0, t = 0

$(1) \Rightarrow \quad -100 = C$

$\therefore \quad A - 100 = -100e^{\frac{-3}{50}t}$

14.The equation of electromotive force for an electric circuit containing resistance and self-inductance is $E = Ri + L\frac{di}{dt}$, where E is the electromotive force is given to the circuit, R the resistance and L, the coefficient of induction. Find the current i at time t when $E = 0$.

Soln: Given $E = Ri + L\frac{di}{dt}$

$\Rightarrow \quad \frac{di}{dt} + \frac{Ri}{L} = \frac{E}{L}$ P=$\frac{R}{L}$, Q=$\frac{E}{L}$

MATHS

Integrating factor, IF= $e^{\int P dt} = e^{\int \frac{R}{L}dt} = e^{\frac{Rt}{L}}$

Complete solution i(IF)= $\int Q(\text{IF})dt + c$

$$i\,(e^{\frac{Rt}{L}}) = \int \frac{E}{L}\left(e^{\frac{Rt}{L}}\right)dt + c \quad \Rightarrow \quad ie^{\frac{Rt}{L}} = \frac{E}{L}\left(\frac{e^{\frac{Rt}{L}}}{\frac{R}{L}}\right) + c$$

$$\Rightarrow i = \frac{E}{R} + ce^{-\frac{Rt}{L}}$$

put E=0 $\Rightarrow$ $i = ce^{-\frac{Rt}{L}}$

1.Matrices and Determinants

1. Find the inverse of the matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

soln: $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $\quad |A| = ad - bc$

$$A^{-1} = \frac{1}{|A|}(\text{adj}A)$$

$$A^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Try yourself:

Find the inverse of(i) $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ **(ii)** $\begin{bmatrix} -2 & 4 \\ 1 & -3 \end{bmatrix}$

2. If A= $\begin{bmatrix} 8 & -4 \\ -5 & 3 \end{bmatrix}$ **,then prove that** $\mathbf{A(adjA) = (adjA)A = |A|I.}$ S-21,S-20

Soln: A= $\begin{bmatrix} 8 & -4 \\ -5 & 3 \end{bmatrix}$ $\Rightarrow$ adjA $= \begin{bmatrix} 3 & 4 \\ 5 & 8 \end{bmatrix}$

$$|A| = 24 - 20 = 4$$

$$|A|I = 4\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$$

$$A(\text{adj}A) = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$$

$$(\text{adj}A)A = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$$

$$A(\text{adj}A) = (\text{adj}A)A = |A|I$$

3. If A= $\begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}$**; B=** $\begin{bmatrix} -1 & -3 \\ 5 & 2 \end{bmatrix}$ **then prove that** $\mathbf{(AB)^{-1} = B^{-1}A^{-1}}$.

Soln: $AB = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}\begin{bmatrix} -1 & -3 \\ 5 & 2 \end{bmatrix} = \begin{bmatrix} 7 & -5 \\ 18 & -11 \end{bmatrix}$

$$|AB| = -77 + 90 = 13$$

$$(AB)^{-1} = \frac{1}{13}\begin{bmatrix} -11 & 5 \\ -18 & 7 \end{bmatrix} \text{.......(1)}$$

$|B| = -2 + 15 = 13 \quad B^{-1} = \frac{1}{13}\begin{bmatrix} 2 & 3 \\ -5 & -1 \end{bmatrix}$

$|A| = 15 - 14 = 1 \quad A^{-1} = \frac{1}{1}\begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}$

$$B^{-1}A^{-1} = \frac{1}{13}\begin{bmatrix} -11 & 5 \\ -18 & 7 \end{bmatrix} \text{.......(2)}$$

$$(AB)^{-1} = B^{-1}A^{-1}$$

Try yourself

If A= $\begin{bmatrix} 0 & -3 \\ 1 & 4 \end{bmatrix}$**; B=** $\begin{bmatrix} -2 & -3 \\ 0 & -1 \end{bmatrix}$ **then prove that** $\mathbf{(AB)^{-1} = B^{-1}A^{-1}}$ **S-20,J-22**

4. If A = $\begin{bmatrix} 2 & 9 \\ 1 & 7 \end{bmatrix}$ **then prove that** $\mathbf{(A^T)^{-1} = (A^{-1})^T}$ **M-20**

Soln:

$A = \begin{bmatrix} 2 & 9 \\ 1 & 7 \end{bmatrix}$ $\lvert A\rvert = 14 - 9 = 5$ $A^{-1} = \frac{1}{5}\begin{bmatrix} 7 & -9 \\ -1 & 2 \end{bmatrix}$ $(A^{-1})^T = \frac{1}{5}\begin{bmatrix} 7 & -1 \\ -9 & 2 \end{bmatrix}$..(1)	$A^T = \begin{bmatrix} 2 & 1 \\ 9 & 7 \end{bmatrix}$ $\lvert A^T\rvert = 14 - 9 = 5$ $(A^T)^{-1} = \frac{1}{5}\begin{bmatrix} 7 & -1 \\ -9 & 2 \end{bmatrix}$..(2)

$$\mathbf{(A^T)^{-1} = (A^{-1})^T}$$

5. S.T. $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ **is an orthogonal matrix. M-23**

Soln: $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ $\quad A^T = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

$$A\,A^T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$$A^T A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$$A\,A^T = A^T A = I$$

A is an orthogonal matrix

6. If A= $\begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$**, find x and y such that** $\mathbf{A^2 + xA + yI = 0}$**. Hence find** $\mathbf{A^{-1}}$**.**

Soln: $x = -[5 + (-2)] = -3$;

$$y = \begin{vmatrix} 5 & 3 \\ -1 & -2 \end{vmatrix} = -10 + 3 = -7$$

$$A^2 - 3A - 7I = 0$$

$$A^2A^{-1} - 3AA^{-1} - 7IA^{-1} = 0$$

$$A - 3I - 7A^{-1} = 0$$

$$A - 3I = 7A^{-1}$$

$$7A^{-1} = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{1}{7}\begin{bmatrix} 2 & 3 \\ -1 & -5 \end{bmatrix}$$

7.If $A\begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} 14 & 7 \\ 7 & 7 \end{bmatrix}$ **then find A.**

Soln: Let $B = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$ $\quad C = \begin{bmatrix} 14 & 7 \\ 7 & 7 \end{bmatrix}$

$$B^{-1} = \frac{1}{-7}\begin{bmatrix} -2 & -3 \\ 1 & 5 \end{bmatrix}$$

$$AB = C \Rightarrow A = CB^{-1}$$

$$= \begin{bmatrix} 14 & 7 \\ 7 & 7 \end{bmatrix}\left(\frac{1}{-7}\right)\begin{bmatrix} -2 & -3 \\ 1 & 5 \end{bmatrix}$$

$$= \frac{1}{-7}\begin{bmatrix} -21 & -7 \\ -7 & 14 \end{bmatrix}$$

$$A = \begin{bmatrix} 3 & 1 \\ 1 & -2 \end{bmatrix}$$

8.If A = $\begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix}$**, B =** $\begin{bmatrix} 3 & -2 \\ 1 & 1 \end{bmatrix}$**, and C =** $\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$**, find** X **such that AXB= C.**

Soln: $A^{-1} = \frac{1}{2}\begin{bmatrix} 0 & 1 \\ -2 & 1 \end{bmatrix}$ $\quad B^{-1} = \frac{1}{5}\begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$

$$AXB = C \Rightarrow X = A^{-1}CB^{-1}$$

$$X = \frac{1}{2}\begin{bmatrix} 0 & 1 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}\frac{1}{5}\begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$$

$$= \frac{1}{10}\begin{bmatrix} 2 & 2 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$$

$$= \frac{1}{10}\begin{bmatrix} 0 & 10 \\ 0 & 0 \end{bmatrix}$$

$$X = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

9. If adj(A) = $\begin{bmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{bmatrix}$ **find** $\mathbf{A^{-1}}$**. J-23**

Soln: $|adjA| = \begin{vmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{vmatrix} = 2(36 - 18)$

$$= 36$$

$$A = \pm\frac{1}{\sqrt{|adjA|}}adjA = \pm\frac{1}{6}\begin{bmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{bmatrix}$$

Try **: adj(A) =** $\begin{bmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ **எனில்,** $\mathbf{A^{-1}}$ **ஐ காண்க.M-25**

10. If adj(A) = $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ **find adj(adjA) .**

Soln: adj(A) = $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix}$

2	0	0	2
0	1	−1	0
0	1	1	0
2	0	0	2

$$\text{adj(adjA)} = \begin{pmatrix} 2 & 0 & 2 \\ 0 & 2 & 0 \\ -2 & 0 & 2 \end{pmatrix}^T = \begin{pmatrix} 2 & 0 & -2 \\ 0 & 2 & 0 \\ 2 & 0 & 2 \end{pmatrix}$$

Try yourself

If $A = \begin{bmatrix} 2 & -1 & 3 \\ -5 & 3 & 1 \\ -3 & 2 & 3 \end{bmatrix}$ **find** $|adj(adjA)|$ **M--24**

11. If $adj(A) = \begin{bmatrix} 2 & -4 & 2 \\ -3 & 12 & -7 \\ -2 & 0 & 2 \end{bmatrix}$ **find A.**

Soln: $adj(A) = \begin{bmatrix} 2 & -4 & 2 \\ -3 & 12 & -7 \\ -2 & 0 & 2 \end{bmatrix}$

$|adjA| = 2(24-0) + 4(-6-14) + 2(0+24)$
$= 48 - 80 + 48$
$= 16$

12	−7	−3	12
0	2	−2	0
−4	2	2	−4
12	−7	−3	12

$adj(adjA) = \begin{pmatrix} 24 & 20 & 24 \\ 8 & 8 & 8 \\ 4 & 8 & 12 \end{pmatrix}^T$
$= \begin{pmatrix} 24 & 8 & 4 \\ 20 & 8 & 8 \\ 24 & 8 & 12 \end{pmatrix}$

$A = \pm \frac{1}{\sqrt{|adjA|}} adj(adjA) = \pm \frac{1}{4} \begin{pmatrix} 24 & 8 & 4 \\ 20 & 8 & 8 \\ 24 & 8 & 12 \end{pmatrix}$
$= \pm \begin{pmatrix} 6 & 2 & 1 \\ 5 & 2 & 2 \\ 6 & 2 & 3 \end{pmatrix}$

12. If $A = \begin{bmatrix} 1 & tanx \\ -tanx & 1 \end{bmatrix}$**, then show that** $A^T A^{-1} = \begin{bmatrix} cos2x & -sin2x \\ sin2x & cos2x \end{bmatrix}$.

Soln: $A^T = \begin{bmatrix} 1 & -tanx \\ tanx & 1 \end{bmatrix}$ $|A| = 1 + \tan^2 x$

$A^{-1} = \frac{1}{sec^2 x} \begin{bmatrix} 1 & -tanx \\ tanx & 1 \end{bmatrix}$

$A^T A^{-1} = \frac{1}{sec^2 x} \begin{bmatrix} 1 & -tanx \\ tanx & 1 \end{bmatrix} \begin{bmatrix} 1 & -tanx \\ tanx & 1 \end{bmatrix}$

$A^T A^{-1} = \begin{bmatrix} cos2x & -sin2x \\ sin2x & cos2x \end{bmatrix}$

13. If $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ **,then show that** $A^{-1} = \frac{1}{2}(A^2 - 3I)$

Soln: $A^3 + xA^2 + yA + zI = 0$
$x = -(0+0+0) = 0;$
$y = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = -3;$
$z = -|A| = -\left[0\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} + 1\begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix}\right]$
$= -[0 - 1(-1) + 1(1)]$
$= -2$
$A^3 + 0A^2 - 3A - 2I = 0$
$A^{-1}(A^3 - 3A - 2I) = 0$
$A^2 - 3I - 2A^{-1} = 0$
$A^2 - 3I = 2A^{-1}$
$A^{-1} = \frac{1}{2}(A^2 - 3I)$

14. If $A = \frac{1}{9}\begin{bmatrix} -8 & 1 & 4 \\ 4 & 4 & 7 \\ 1 & -8 & 4 \end{bmatrix}$**, Show that** $A^{-1} = A^T$.

Soln: $A = \frac{1}{9}\begin{bmatrix} -8 & 1 & 4 \\ 4 & 4 & 7 \\ 1 & -8 & 4 \end{bmatrix}$ $A^T = \frac{1}{9}\begin{bmatrix} -8 & 4 & 1 \\ 1 & 4 & -8 \\ 4 & 7 & 4 \end{bmatrix}$

$AA^T = \frac{1}{81}\begin{bmatrix} 81 & 0 & 0 \\ 0 & 81 & 0 \\ 0 & 0 & 81 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$
$AA^T = I$
$A^T = A^{-1}$

15. If $F(\alpha) = \begin{bmatrix} cos\alpha & 0 & sin\alpha \\ 0 & 1 & 0 \\ -sin\alpha & 0 & cos\alpha \end{bmatrix}$ **S.T.** $[F(\alpha)]^{-1} = F(-\alpha)$.

Soln: $F(\alpha) = \begin{bmatrix} cos\alpha & 0 & sin\alpha \\ 0 & 1 & 0 \\ -sin\alpha & 0 & cos\alpha \end{bmatrix}$ M-23

$F(-\alpha) = \begin{bmatrix} cos\alpha & 0 & -sin\alpha \\ 0 & 1 & 0 \\ sin\alpha & 0 & cos\alpha \end{bmatrix}$

$F(\alpha)\,F(-\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$
$F(\alpha)\,F(-\alpha) = I$
$F(-\alpha) = [F(\alpha)]^{-1}$

16. Find the rank of the matrix

(i) $\begin{bmatrix} 1 & 1 & 1 & 3 \\ 2 & -1 & 3 & 4 \\ 5 & -1 & 7 & 11 \end{bmatrix}$ **(ii)** $\begin{bmatrix} 2 & -2 & 4 & 3 \\ -3 & 4 & -2 & -1 \\ 6 & 2 & -1 & 7 \end{bmatrix}$

Soln: (i) $A = \begin{bmatrix} 1 & 1 & 1 & 3 \\ 2 & -1 & 3 & 4 \\ 5 & -1 & 7 & 11 \end{bmatrix}$

$\sim \begin{bmatrix} 1 & 1 & 1 & 3 \\ 0 & -3 & 1 & -2 \\ 0 & -6 & 2 & -4 \end{bmatrix}$ $R_2 \to R_2 - 2R_1;\ R_3 \to R_3 - 5R_1$

$\sim \begin{bmatrix} 1 & 1 & 1 & 3 \\ 0 & -3 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $R_3 \to R_3 - 2R_2$

The number of non-zero row's is 2 $\therefore \rho(A) = 2$

(ii) $A = \begin{bmatrix} 2 & -2 & 4 & 3 \\ -3 & 4 & -2 & -1 \\ 6 & 2 & -1 & 7 \end{bmatrix}$

$\sim \begin{bmatrix} 2 & -2 & 4 & 3 \\ 0 & 2 & 8 & 7 \\ 0 & 8 & -13 & -2 \end{bmatrix}$ $R_2 \to 2R_2 + 3R_1;\ R_3 \to R_3 - 3R_1$

$\sim \begin{bmatrix} 1 & 1 & 1 & 3 \\ 0 & -3 & 1 & -2 \\ 0 & 0 & -45 & -30 \end{bmatrix}$ $R_3 \to R_3 - 4R_2$

The number of non-zero row's is 3 $\therefore \rho(A) = 3$

17.Find rank of the matrix by minor method

(i) $\begin{bmatrix} 1 & -2 & 3 \\ 2 & 4 & -6 \\ 5 & 1 & -1 \end{bmatrix}$ **J-22** **(ii)** $\begin{bmatrix} 3 & 2 & 5 \\ 1 & 1 & 2 \\ 3 & 3 & 6 \end{bmatrix}$

Soln:

$|A| = \begin{vmatrix} 1 & -2 & 3 \\ 2 & 4 & -6 \\ 5 & 1 & -1 \end{vmatrix}$
$= 1(-4+6) + 2(-2+30) + 3(2-20)$
$= 2 + 56 - 54$
$= 4 \neq 0$ $\therefore \rho(A) = 3$

(ii) $|A| = \begin{vmatrix} 3 & 2 & 5 \\ 1 & 1 & 2 \\ 3 & 3 & 6 \end{vmatrix}$
$= 3(6-6) - 2(6-6) + 5(3-3) = 0$
$\therefore \rho(A) < 3$

Consider 2×2 minor $\begin{vmatrix} 3 & 2 \\ 1 & 1 \end{vmatrix} = 3 - 2 = 1 \neq 0$
$\therefore \rho(A) = 2$

Try yourself: **Find the rank (i)** $\begin{bmatrix} 0 & 1 & 2 & 1 \\ 0 & 2 & 4 & 3 \\ 8 & 1 & 0 & 2 \end{bmatrix}$ **s-21** **(ii)** $\begin{bmatrix} 1 & 2 & -1 \\ 3 & -1 & 2 \\ 1 & -2 & 3 \\ 1 & -1 & 1 \end{bmatrix}$ **m-22** **(iii)** $\begin{bmatrix} -1 & 3 \\ 4 & -7 \\ 3 & -4 \end{bmatrix}$ **m-24**

18.Solve by inversion method $5x + 2y = 3, 3x + 2y = 5$.

Soln: **M-22**

$\begin{pmatrix} 5 & 2 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$

$A = \begin{pmatrix} 5 & 2 \\ 3 & 2 \end{pmatrix}, X = \begin{pmatrix} x \\ y \end{pmatrix}, B = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$

$A^{-1} = \frac{1}{4}\begin{pmatrix} 2 & -2 \\ -3 & 5 \end{pmatrix}$

$X = A^{-1}B$

$$\begin{pmatrix}x\\y\end{pmatrix}=\frac{1}{4}\begin{pmatrix}2&-2\\-3&5\end{pmatrix}\begin{pmatrix}3\\5\end{pmatrix}$$

$$\begin{pmatrix}x\\y\end{pmatrix}=\frac{1}{4}\begin{pmatrix}-4\\16\end{pmatrix}$$

$$\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}-1\\4\end{pmatrix}$$

Try yourself:Solve $2x+5y=-2, x+2y=-3$ M-25

19.Solve by cramer's method $\frac{3}{x}+2y=12$, $\frac{2}{x}+3y=13$.

Soln:$\Delta=\begin{vmatrix}3&2\\2&3\end{vmatrix}=9-4=5$

$\Delta_1=\begin{vmatrix}12&2\\13&3\end{vmatrix}=36-26=10$

$\Delta_2=\begin{vmatrix}3&12\\2&13\end{vmatrix}=39-24=15$

$\frac{1}{x}=\frac{\Delta_1}{\Delta}=\frac{10}{5}=2$

$y=\frac{\Delta_2}{\Delta}=\frac{15}{5}=3$

$x=\frac{1}{2},\quad y=3$

20. Solve the system of equations $x+2y+3z=0$; $3x+4y+4z=0$; $7x+10y+12z=0$

Soln: $(A|B)=\left(\begin{array}{ccc|c}1&2&3&0\\3&4&4&0\\7&10&12&0\end{array}\right)$

$\sim\left(\begin{array}{ccc|c}1&2&3&0\\0&-2&-5&0\\0&-4&-9&0\end{array}\right)$ $R_2\to R_2-3R_1$; $R_3\to R_3-7R_1$

$\sim\left(\begin{array}{ccc|c}1&2&3&0\\0&-2&-5&0\\0&0&1&0\end{array}\right)$ $R_3\to R_3-2R_2$

The system has only Trivial solution

$x=0,\quad y=0,\quad z=0$

21. $A=\begin{bmatrix}0&5\\-1&6\end{bmatrix}$ find inverse of non singular matrix by using gauss jordan method.

Soln: $(A|I)=\left(\begin{array}{cc|cc}0&5&1&0\\-1&6&0&1\end{array}\right)$

$=\left(\begin{array}{cc|cc}-1&6&0&1\\0&5&1&0\end{array}\right)$ $R_1\leftrightarrow R_2$

$=\left(\begin{array}{cc|cc}1&-6&0&-1\\0&1&1/5&0\end{array}\right)$ $R_1\to(-1)R_1$; $R_2\to\frac{1}{5}R_2$

$=\left(\begin{array}{cc|cc}1&0&6/5&-1\\0&1&1/5&0\end{array}\right)$ $R_1\to R_1+6R_2$

$A^{-1}=\begin{bmatrix}6/5&-1\\1/5&0\end{bmatrix}$

$A^{-1}=\frac{1}{5}\begin{bmatrix}6&-5\\1&0\end{bmatrix}$

22. Solve the following system of linear equations by matrix inversion method: $x+y+z-2=0$; $6x-4y+5z-31=0$; $5x+2y+2z=13$

Soln: $\begin{pmatrix}1&1&1\\6&-4&5\\5&2&2\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}2\\31\\13\end{pmatrix}$

Let $A=\begin{pmatrix}1&1&1\\6&-4&5\\5&2&2\end{pmatrix}$, $X=\begin{pmatrix}x\\y\\z\end{pmatrix}$, $B=\begin{pmatrix}2\\31\\13\end{pmatrix}$

$|A|=\begin{vmatrix}1&1&1\\6&-4&5\\5&2&2\end{vmatrix}$

$=1(-8-10)-1(12-25)+1(12+20)$

$=-18+13+32$

$=27\neq0\ \therefore A^{-1}\ exist$

$adjA=\begin{pmatrix}(-8-10)&(25-12)&(12+20)\\(2-2)&(2-5)&(5-2)\\(5+4)&(6-5)&(-4-6)\end{pmatrix}^T$

−4	5	6	−4
2	2	5	2
1	1	1	1
−4	5	6	−4

$=\begin{pmatrix}-18&13&32\\0&-3&3\\9&1&-10\end{pmatrix}^T$

$=\begin{pmatrix}-18&0&9\\13&-3&1\\32&3&-10\end{pmatrix}$

$A^{-1}=\frac{1}{|A|}(adjA)=\frac{1}{27}\begin{pmatrix}-18&0&9\\13&-3&1\\32&3&-10\end{pmatrix}$

$X=A^{-1}B=\frac{1}{27}\begin{pmatrix}-18&0&9\\13&-3&1\\32&3&-10\end{pmatrix}\begin{pmatrix}2\\31\\13\end{pmatrix}$

$=\frac{1}{27}\begin{pmatrix}-36+0+117\\26-93+13\\64+93-130\end{pmatrix}$

$=\frac{1}{27}\begin{pmatrix}81\\-54\\27\end{pmatrix}$

$\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}3\\-2\\1\end{pmatrix}$

Try yourself

$2x_1+3x_2+3x_3=5$, $x_1-2x_2+x_3=-4$, $3x_1-x_2-2x_3=3$

23.If $A=\begin{bmatrix}-4&4&4\\-7&1&3\\5&-3&-1\end{bmatrix}$ and $B=\begin{bmatrix}1&-1&1\\1&-2&-2\\2&1&3\end{bmatrix}$, find the products AB and BA and hence solve the equations $x-y+z=4$; $x-2y-2z=9$; $2x+y+3z=1$

Soln: $AB=\begin{bmatrix}8&0&0\\0&8&0\\0&0&8\end{bmatrix}=8I_3$

$AB=8I_3$

Similarly $BA=8I_3$

$B^{-1}=\frac{1}{8}A$

$\therefore B^{-1}=\frac{1}{8}\begin{bmatrix}-4&4&4\\-7&1&3\\5&-3&-1\end{bmatrix}$

Given system of equation can be written into matrix form

$\begin{pmatrix}1&-1&1\\1&-2&-2\\2&1&3\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}4\\9\\1\end{pmatrix}$

$B\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}4\\9\\1\end{pmatrix}$

$\begin{pmatrix}x\\y\\z\end{pmatrix}=B^{-1}\begin{pmatrix}4\\9\\1\end{pmatrix}$

$\begin{pmatrix}x\\y\\z\end{pmatrix}=\frac{1}{8}\begin{bmatrix}-4&4&4\\-7&1&3\\5&-3&-1\end{bmatrix}\begin{pmatrix}4\\9\\1\end{pmatrix}$

$=\frac{1}{8}\begin{pmatrix}24\\-16\\-8\end{pmatrix}$

$\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}3\\-2\\-1\end{pmatrix}$

24. Solve the linear equations by Cramer's rule: J-22

$3x+3y-z=11$; $2x-y+2z=9$; $4x+3y+2z=25$

Soln:

$$\Delta = \begin{vmatrix} 3 & 3 & -1 \\ 2 & -1 & 2 \\ 4 & 3 & 2 \end{vmatrix}$$

$= 3(-2-6) - 3(4-8) - 1(6+4)$

$= -24 + 12 - 10 = -22$

$$\Delta_1 = \begin{vmatrix} 11 & 3 & -1 \\ 9 & -1 & 2 \\ 25 & 3 & 2 \end{vmatrix}$$

$= 11(-2-6) - 3(18-50) - 1(27+25)$

$= -88 + 96 - 52 = -44$

$$\Delta_2 = \begin{vmatrix} 3 & 11 & -1 \\ 2 & 9 & 2 \\ 4 & 25 & 2 \end{vmatrix}$$

$= 3(18-50) - 11(4-8) - 1(50-36)$

$= -96 + 44 - 14 = -66$

$$\Delta_3 = \begin{vmatrix} 3 & 3 & 11 \\ 2 & -1 & 9 \\ 4 & 3 & 25 \end{vmatrix}$$

$= 3(-25-27) - 3(50-36) + 11(6+4)$

$= -156 - 42 + 110 = -88$

Cramer's rule: $x = \frac{\Delta_1}{\Delta} = \frac{-44}{-22} = 2$

$y = \frac{\Delta_2}{\Delta} = \frac{-66}{-22} = 3$

$z = \frac{\Delta_3}{\Delta} = \frac{-88}{-22} = 4$

Try Yourself

(i) $\frac{3}{x} - \frac{4}{y} - \frac{2}{z} = 1, \frac{1}{x} + \frac{2}{y} + \frac{1}{z} = 2, \frac{2}{x} - \frac{5}{y} - \frac{4}{z} = -1$ **M-24**

(ii) $x_1 - x_2 = 3,\ 2x_1 + 3x_2 + 4x_3 = 17,\ x_2 + 2x_3 = 7$ J-23

(iii) $3x + y + z = 2\ ; x - 3y + 2z = 1; 7x - y + 4z = 5$ M-22

Cramer's rule not applicable why?

(iv) $x - y + 2z = 2\ ;\ 2x + y + 4z = 7;\ 4x - y + z = 4$ S-21

25. Solve by Gaussian elimination method:

$-2y + 3z = 2\ ; x + 2y - z = 3;\ 3x - y + 2z = 1.$

Soln: $(A|B) = \left(\begin{array}{ccc|c} 2 & -2 & 3 & 2 \\ 1 & 2 & -1 & 3 \\ 3 & -1 & 2 & 1 \end{array}\right)$

$\sim \left(\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 2 & -2 & 3 & 2 \\ 3 & -1 & 2 & 1 \end{array}\right)$ $R_1 \leftrightarrow R_2$

$\sim \left(\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & -6 & 5 & -4 \\ 0 & -7 & 5 & -8 \end{array}\right)$ $R_2 \to R_2 - 2R_1;$

$R_3 \to R_3 - 3R_1$

$\sim \left(\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & -6 & 5 & -4 \\ 0 & 0 & -5 & -20 \end{array}\right)$ $R_3 \to 6R_3 - 7R_2$

$-5z = -20$ $z = 4$	$-6y + 5z = -4$ $y = 4$	$x + 2y - z = 3$ $x = -1$

<u>**Try yourself**</u>

1. A boy is walking along the path $y = ax^2 + bx + c$ through the points $(-6, 8), (-2, -12)$, and $(3, 8)$. He wants to meet his friend at $P(7, 60)$. Will he meet his friend? (Use Gaussian elimination method.) M-23

2. If $ax^2 + bx + c$ is divided by $x + 3, x - 5$, and $x - 1$, the remainders are $21, 61$ and 9 respectively. Find a, b and c. (Use Gaussian elimination method.)

26.Test for consistency and if possible, solve the following systems of equations by rank method.

(i) $x - y + 2z = 2\ ; 2x + y + 4z = 7\ ;\ 4x - y + z = 4.$

Soln: $(A|B) = \left(\begin{array}{ccc|c} 1 & -1 & 2 & 2 \\ 2 & 1 & 4 & 7 \\ 4 & -1 & 1 & 4 \end{array}\right)$

$\sim \left(\begin{array}{ccc|c} 1 & -1 & 2 & 2 \\ 0 & 3 & 0 & 3 \\ 0 & 3 & -7 & -4 \end{array}\right)$ $R_2 \to R_2 - 2R_1;$

$R_3 \to R_3 - 4R_1$

$\sim \left(\begin{array}{ccc|c} 1 & -1 & 2 & 2 \\ 0 & 3 & 0 & 3 \\ 0 & 0 & -7 & -7 \end{array}\right)$ $R_3 \to R_3 - R_2$

$\rho(A) = \rho(A|B) = 3,\ unknowns = 3$

∴ The system is consistent and has unique solution

$-7z = -7$ $z = 1$	$3y = 3$ $y = 1$	$x - y + 2z = 2$ $x = 1$

(ii) $3x + y + z = 2\ ; x - 3y + 2z = 1\ ;\ 7x - y + 4z = 5.$

Soln: $(A|B) = \left(\begin{array}{ccc|c} 3 & 1 & 1 & 2 \\ 1 & -3 & 2 & 1 \\ 7 & -1 & 4 & 5 \end{array}\right)$

$\sim \left(\begin{array}{ccc|c} 1 & -3 & 2 & 1 \\ 3 & 1 & 1 & 2 \\ 7 & -1 & 4 & 5 \end{array}\right)$ $R_1 \leftrightarrow R_2$

$\sim \left(\begin{array}{ccc|c} 1 & -3 & 2 & 1 \\ 0 & 10 & -5 & -1 \\ 0 & 20 & -10 & -2 \end{array}\right)$ $R_2 \to R_2 - 3R_1;$

$R_3 \to R_3 - 7R_1$

$\sim \left(\begin{array}{ccc|c} 1 & -3 & 2 & 1 \\ 0 & 10 & -5 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right)$ $R_3 \to R_3 - 2R_2$

$\rho(A) = \rho(A|B) = 2,\quad unknowns = 3$

∴ The system is consistent & Infinitely many solution

Put $z = t$ $\forall\, t \in R$	$10y - 5z = -1$ $y = \frac{1}{10}(5t - 1)$	$x - 3y + 2z = 1$ $x = \frac{1}{10}(7 - 5t)$

(iii) $2x - y + z = 2\ ; 6x - 3y + 3z = 6\ ;\ 4x - 2y + 2z = 4.$

Soln: $(A|B) = \left(\begin{array}{ccc|c} 2 & -1 & 1 & 2 \\ 6 & -3 & 3 & 6 \\ 4 & -2 & 2 & 4 \end{array}\right)$

$\sim \left(\begin{array}{ccc|c} 2 & -1 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$ $R_2 \to R_2 - 3R_1;$

$R_3 \to R_3 - 2R_1$

$\rho(A) = \rho(A|B) = 1,\quad unknowns = 3$

∴ The system is consistent & Infinitely many solutions.

Put $z = t$ $\forall\, t \in R$	$y = s$ $\forall\, s \in R$	$2x - y + z = 2$ $x = \frac{1}{2}(2 + s - t)$

(iv) $2x + 2y + z = 5\ ; x - y + z = 1\ ;\ 3x + y + 2z = 4.$

Soln: $(A|B) = \left(\begin{array}{ccc|c} 2 & 2 & 1 & 5 \\ 1 & -1 & 1 & 1 \\ 3 & 1 & 2 & 4 \end{array}\right)$ S-20

$\sim \left(\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 2 & 2 & 1 & 5 \\ 3 & 1 & 2 & 4 \end{array}\right)$ $R_1 \leftrightarrow R_2$

$\sim \left(\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 0 & 4 & -1 & 3 \\ 0 & 4 & -1 & 1 \end{array}\right)$ $R_2 \to R_2 - 2R_1;$

$R_3 \to R_3 - 3R_1$

$\sim \left(\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 0 & 4 & -1 & 3 \\ 0 & 0 & 0 & -2 \end{array}\right)$ $R_3 \to R_3 - R_2$

$\rho(A) = 2 \qquad \rho(A|B) = 3$

$\rho(A) \neq \rho(A|B)$

∴ The system is Inconsistent and has no solution.

27. Find the value of k for which the equations

$kx - 2y + z = 1\ ; x - 2ky + z = -2\ ;\ x - 2y + kz = 1.$

have (i) no solution (ii) a unique solution

(iii) infinitely many solution

Soln:

$$(A|B)=\left(\begin{array}{ccc|c} k & -2 & 1 & 1\\ 1 & -2k & 1 & -2\\ 1 & -2 & k & 1\end{array}\right)$$

$$\sim\left(\begin{array}{ccc|c} 1 & -2 & k & 1\\ 1 & -2k & 1 & -2\\ k & -2 & 1 & 1\end{array}\right)\quad R_1 \leftrightarrow R_3$$

$$\sim\left(\begin{array}{ccc|c} 1 & -2 & k & 1\\ 0 & -2k+2 & 1-k & -3\\ 0 & -2+2k & 1-k^2 & 1-k\end{array}\right)\quad R_2 \to R_2 - R_1;$$

$$R_3 \to R_3 - kR_1$$

$$\sim\left(\begin{array}{ccc|c} 1 & -2 & k & 1\\ 0 & -2k+2 & 1-k & -3\\ 0 & 0 & -k^2-k+2 & -2-k\end{array}\right)\quad R_3 \to R_3 + R_2$$

$$\sim\left(\begin{array}{ccc|c} 1 & -2 & k & 1\\ 0 & -2k+2 & 1-k & -3\\ 0 & 0 & (1-k)(2+k) & -2-k\end{array}\right)$$

(i) No solution: when $k = 1$

$\rho(A) = 2$ & $\rho(A|B) = 3 \Rightarrow \rho(A) \neq \rho(A|B)$

∴ The system is inconsistent and has no solution.

(ii) Infinitely many solutions: when $k = -2$

$\rho(A) = \rho(A|B) = 2$

∴ The system is consistent & infinitely many solution.

(iii) Unique solution: when $k \neq 1$ and $k \neq -2$

$\rho(A) = \rho(A|B) = 3$

∴ The system is consistent & unique solution.

Try yourself: $x - y + z = -9$; $2x - y + z = 4$; $3x - y + z = 6$; $4x - y + 2z = 7$ **m-20**

28. Investigate the values of λ and μ $2x + 3y + 5z = 9$, $7x + 3y - 5z = 8$, $2x + 3y + \lambda z = \mu$ have (i) no solution (ii) a unique solution (iii) an infinite number of solutions.

Soln: $(A|B)=\left(\begin{array}{ccc|c} 2 & 3 & 5 & 9\\ 7 & 3 & -5 & 8\\ 2 & 3 & \lambda & \mu\end{array}\right)$

$$\sim\left(\begin{array}{ccc|c} 2 & 3 & 5 & 9\\ 0 & -15 & -45 & -47\\ 0 & 0 & \lambda-5 & \mu-9\end{array}\right)\quad R_2 \to 2R_2 - 7R_1;$$

$$R_3 \to R_3 - R_1$$

(i) No solution: When $\lambda - 5 = 0$ and $\mu - 9 \neq 0$

$\rho(A) = 2,\ \rho(A|B) = 3$

$\rho(A) \neq \rho(A|B)$

∴ The system is inconsistent and no solution.

(ii) Infinite solutions: When $\lambda - 5 = 0$ and $\mu - 9 = 0$

$\rho(A) = \rho(A|B) = 2$

∴ The system is consistent & infinitely many solution.

(iii) Unique solution: When $\lambda - 5 \neq 0$ and $\mu - 9 \neq 0$

$\rho(A) = \rho(A|B) = 3$

∴ The system is consistent and has unique solution.

29. Determine the values of λ

$x + y + 3z = 0$; $4x + 3y + \lambda z = 0$; $2x + y + 2z = 0$ has (i) a unique solution (ii) a non-trivial solution.

Soln: $(A|B)=\left(\begin{array}{ccc|c} 1 & 1 & 3 & 0\\ 4 & 3 & \lambda & 0\\ 2 & 1 & 2 & 0\end{array}\right)$

$$\sim\left(\begin{array}{ccc|c} 1 & 1 & 3 & 0\\ 2 & 1 & 2 & 0\\ 4 & 3 & \lambda & 0\end{array}\right)\quad R_2 \leftrightarrow R_3$$

$$\sim\left(\begin{array}{ccc|c} 1 & 1 & 3 & 0\\ 0 & -1 & -4 & 0\\ 0 & -1 & \lambda-12 & 0\end{array}\right)\quad R_2 \to R_2 - 2R_1;$$

$$R_3 \to R_3 - 4R_1$$

$$\sim\left(\begin{array}{ccc|c} 1 & 1 & 3 & 0\\ 0 & -1 & -4 & 0\\ 0 & 0 & \lambda-8 & 0\end{array}\right)\quad R_3 \to R_3 - R_2$$

(i) Trivial solution (unique) when $\lambda - 8 \neq 0$

$\rho(A) = \rho(A|B) = 3$

$x = 0,\quad y = 0,\quad z = 0$

(ii) Non-trivial solution when $\lambda - 8 = 0$

$\rho(A) = \rho(A|B) = 2$

30. By using Gaussian elimination method, balance the chemical reaction equation: $C_2H_6 + O_2 \to H_2O + CO_2$

Soln: $x_1\, C_2H_6 + x_2\, O_2 \to x_3\, H_2O + x_4\, CO_2$

Equating C, H and O atoms on both sides

$2x_1 = x_4 \quad \Rightarrow 2x_1 + 0x_2 - 0x_3 - x_4 = 0$

$6x_1 = 2x_3 \quad \Rightarrow 6x_1 + 0x_2 - 2x_3 - 0x_4 = 0$

$2x_2 = x_3 + 2x_4 \Rightarrow 0x_1 + 2x_2 - x_3 - 2x_4 = 0$

$$(A|B)=\left(\begin{array}{cccc|c} 2 & 0 & 0 & -1 & 0\\ 6 & 0 & -2 & 0 & 0\\ 0 & 2 & -1 & -2 & 0\end{array}\right)$$

Take $\boldsymbol{x_4 = 4}$

(2) $\Rightarrow \boldsymbol{x_1 = 2}$

(1) $\Rightarrow \boldsymbol{x_3 = 6}$

(3) $\Rightarrow \boldsymbol{x_2 = 7}$

∴ $\boldsymbol{2C_2H_6 + 7O_2 \to 6H_2O + 4CO_2}$

31. By using Gaussian elimination method, balance the chemical reaction equation $C_5H_8 + O_2 \to CO_2 + H_2O$

Solu:

$x_1C_5H_8 + x_2O_2 \to x_3CO_2 + x_4H_2O$

Equating C, H and O atoms on both sides

$C \Rightarrow \boldsymbol{5x_1 = x_3 ---\to (1)}$

$H \Rightarrow \boldsymbol{8x_1 = 2x_4 --\to (2)}$

$O \Rightarrow \boldsymbol{8x_2 = 2x_3 + x_4 --\to (3)}$

$$\boldsymbol{(A|B)}=\left(\begin{array}{cccc|c} 5 & 0 & -1 & 0 & 0\\ 8 & 0 & 0 & -2 & 0\\ 0 & 2 & -2 & -1 & 0\end{array}\right)$$

Take $\boldsymbol{x_4 = 4}$

(2) $\Rightarrow \boldsymbol{x_1 = 1}$

(1) $\Rightarrow \boldsymbol{x_3 = 5}$

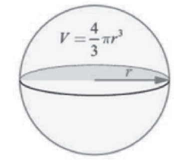
(3) $\Rightarrow \boldsymbol{x_2 = 7}$

∴ $\boldsymbol{1C_5H_8 + 7O_2 \to 5CO_2 + 4H_2O}$.

DIFFERENTIAL CALCULUS(FIRST PART)

1. If we blow air into a balloon of spherical shape at a rate of 1000 cm^3 per second. At what rate the radius of the baloon changes when the radius is 7cm? Also compute the rate at which the surface area changes.

Soln: Given $\frac{dV}{dt} = 1000,\quad r = 7$



Volume of sphere $V = \frac{4}{3}\pi r^3$

d.w.r.t 't' $\frac{dV}{dt} = \frac{4}{3}\pi(3r^2)\frac{dr}{dt}$

At $r = 7$, $\quad 1000 = 4\pi(7)^2\frac{dr}{dt}$

$\frac{1000}{4\times 49\pi} = \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{250}{49\pi}\, cm/sec$

Surface area $S = 4\pi r^2$

d.w.r.t 't' $\frac{dS}{dt} = 4\pi(2r)\frac{dr}{dt}$

At $r = 7$, $\frac{dS}{dt} = 8\pi(7)\left(\frac{250}{49}\right)$

$\frac{dS}{dt} = \frac{2000}{7} cm^2/sec$

2. Salt is poured from a conveyer belt at a rate of 30 cubic metre per minute forming a conical pile with a circular base whose height and diameter of base are always equal. How fast is the height of the pile increasing when the pile is 10 metre high?

Soln: Given $\frac{dV}{dt} = 30$, $h = 2r \ (or) r = \frac{h}{2}$, $h = 10$

Volume of cone $V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h$

$V = \frac{\pi}{12}h^3$

d.w.r.t 't' $\frac{dV}{dt} = \frac{\pi}{12}(3h^2)\frac{dh}{dt}$

at $h = 10$, $30 = \frac{\pi}{4}(10)^2\frac{dh}{dt}$

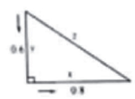
$\frac{30\times4}{100\pi} = \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{6}{5\pi} \ m/min$

3. A road running north to south crosses a road going east to west at the point P. Car A is driving north along the first road, and car B is driving east along the second road. At a particular time car A is 10 kilometres to the north of P and traveling at 80 km/hr, while car B is 15 kilometres to the east of P and traveling at 100 km/hr. How fast is the distance between the two cars changing?

Soln: Given $x = 10$, $y = 15$,

$\frac{dx}{dt} = 80$, $\frac{dy}{dt} = 100$



$z^2 = x^2 + y^2$	$z^2 = x^2 + y^2$
$z^2 = (10)^2 + (15)^2$	$z\frac{dz}{dt} = x\frac{dx}{dt} + y\frac{dy}{dt}$
$z^2 = 100 + 225$	$5\sqrt{13}\frac{dz}{dt} = 10(80) + 15(100)$
$z = \sqrt{325} = 5\sqrt{13}$	$\frac{dz}{dt} = \frac{2300}{5\sqrt{13}} = \frac{460}{\sqrt{13}} \ km/hr$

4. A conical water tank with vertex down of 12 metres height has a radius of 5 metres at the top. If water flows into the tank at a rate 10 cubic m/min, how fast is the depth of the water increases when the water is 8 metres deep? M-24

Soln: Given $h = 12$, $r = 5$, $\frac{dV}{dt} = 10$

From similar triangle $\frac{r}{h} = \frac{5}{12} \implies r = \frac{5h}{12}$

Volume of cone $V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi\left(\frac{5h}{12}\right)^2 h$

$V = \frac{25}{3\times144}\pi h^3$

d.w.r.t 't' $\frac{dz}{dt} = \frac{25}{3\times144}\pi(3h^2)\frac{dh}{dt}$

$\frac{dz}{dt} = \frac{25}{144}\pi(h^2)\frac{dh}{dt}$

at $h = 8$, $10 = \frac{25}{144}\pi(8)^2\frac{dh}{dt}$

$\frac{dh}{dt} = \frac{144\times10}{64\times25}$

$\frac{dh}{dt} = \frac{9}{10\pi} m/min$

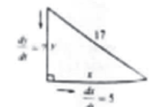
5. A ladder 17 metre long is leaning against the wall. The base of the ladder is pulled away from the wall at a rate of 5 m/s. When the base of the ladder is 8 metres from the wall.(i) How fast is the top of the ladder moving down the wall?(ii) At what rate, the area of the triangle formed by the ladder, wall, and the floor, is changing?

Soln: Given $x = 8$, $\frac{dx}{dt} = 5$

$17^2 = x^2 + y^2$	$0 = x\frac{dx}{dt} + y\frac{dy}{dt}$
$17^2 = (8)^2 + y^2$	$0 = 8(5) + 15\frac{dy}{dt}$
$289 = 64 + y^2$	$\frac{dy}{dt} = \frac{-40}{15} = \frac{-8}{3} \ m/Sec$

$y^2 = 225$ $\quad y = 15$

Area of triangle $A = \frac{1}{2}xy$



$\frac{dA}{dt} = \frac{1}{2}\left[x\frac{dy}{dt} + y\frac{dx}{dt}\right]$

$\frac{dA}{dt} = \frac{1}{2}\left[8\left(\frac{-8}{3}\right) + 15\times5\right]$

$\frac{dA}{dt} = \frac{1}{2}\left[\frac{-64}{3} + 75\right]$ $\quad \frac{dA}{dt} = \frac{161}{6} m^2/s$

6. A police jeep, approaching an orthogonal intersection from the northern direction, is chasing a speeding car that has turned and moving straight east. When the jeep is 0.6 km north of the intersection and the car is 0.8 km to the east. The police determine with a radar that the distance between them and the car is increasing at 20 km/hr. If the jeep is moving at 60 km/hr at the instant of measurement, what is the speed of the car? M-20

Soln: Given $x = 0.8$, $y = 0.6$,

$\frac{dz}{dt} = 20$, $\frac{dy}{dt} = -60$

$z^2 = x^2 + y^2$	$z^2 = x^2 + y^2$
$z^2 = (0.8)^2 + (0.6)^2$	$z\frac{dz}{dt} = x\frac{dx}{dt} + y\frac{dy}{dt}$
$z^2 = 0.64 + 0.36$	$1(20) = 0.8\left(\frac{dx}{dt}\right) + 0.6(-60)$
$z^2 = 1 \implies z = 1$	$\frac{dx}{dt} = \frac{20+3}{0.8} = 70 \ km/hr$

7. Find the acute angle b/w $y = x^2$ and $y = (x-3)^2$.

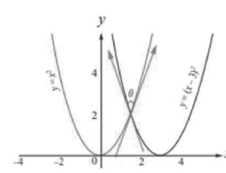
Soln: Given $y = x^2$ and $y = (x-3)^2$ M-24

$x^2 = (x-3)^2$

$x^2 = x^2 - 6x + 9$

$-6x + 9 = 0$

$x = \frac{3}{2} \implies y = \frac{9}{4}$



$\therefore$ the point of intersection $\left(\frac{3}{2}, \frac{9}{4}\right)$

$m_1 = \frac{dy}{dx} = 2x$, $\quad m_2 = \frac{dy}{dx} = 2(x-3)$

$m_1 = 3$, $\quad m_2 = -3$

angle between the curve $\theta = tan^{-1}\left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$= tan^{-1}\left|\frac{3+3}{1-9}\right|$

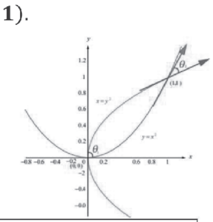
$= tan^{-1}\left(\frac{3}{4}\right)$

8.Find the angle between the curves $y = x^2$ and $x = y^2$ at their point of intersections $(0,0)$ and $(1,1)$.

Soln: $\frac{dy}{dx} = 2x$, $\quad \frac{dy}{dx} = \frac{1}{2y}$

Angle between the curve

$\theta = tan^{-1}\left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$



At$(0,0)$ $m_1 = 0$ $m_2 = \infty$	At$(1,1)$ $m_1 = 2$ $m_2 = \frac{1}{2}$
$\theta = tan^{-1}\left\|\frac{0-\infty}{1+(0)(\infty)}\right\|$	$\theta = tan^{-1}\left\|\frac{2-\frac{1}{2}}{1+(2)\left(\frac{1}{2}\right)}\right\|$
$\theta = \frac{\pi}{2}$	$\theta = tan^{-1}\left\|\frac{3/2}{2}\right\|$
	$\theta = tan^{-1}\left(\frac{3}{4}\right)$ M-22

9. Find the angle between the rectangular hyperbola $xy = 2$ and the parabola $x^2y + 4 = 0$.

Soln: $xy = 2 \implies y = \frac{2}{x}$ and $x^2y + 4 = 0$

$x^2\left(\frac{2}{x}\right) + 4 = 0$

$2x + 4 = 0$

$x = -2$ i.e, $y = -1$

$\therefore$ the point of intersection $(-2, -1)$

$m_1 = \frac{dy}{dx} = \frac{-2}{x^2}$, $\quad m_2 = \frac{dy}{dx} = \frac{-4(-2)}{x^3}$

$m_1 = \frac{-1}{2}$, $\quad m_2 = -1$

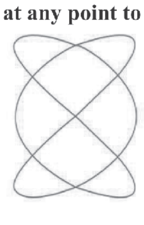
angle between the curve $\theta = tan^{-1}\left|\frac{m_1-m_2}{1+m_1m_2}\right|$

$= tan^{-1}\left|\frac{\frac{-1}{2}-1}{1-\left(\frac{-1}{2}\right)\times -1}\right|$

$= tan^{-1}(3)$

10. Find the equation of tangent and normal at any point to the Lissajous curve given by

$x = 2cos3t$ and $y = 3sin2t, \; t\epsilon R$



Soln: $x = 2cos3t \quad y = 3sin2t$

$m = \frac{dy}{dx} = -\frac{3\,cos2t \times 2d}{2sin3 \times 3d} = -\frac{cos2t}{sin3t}$

Equation of tangent

$(y - y_1) = m(x - x_1)$

$y - 3sin2t = -\frac{cos2t}{sin3t}(x - 2cos3t)$

$xcos2t + ysin3t = 3sin2tsin3t + 2cos2tcos3t$

Equation of normal $(y - y_1) = -\frac{1}{m}(x - x_1)$

$y - 3sin2t = \frac{sin3t}{cos2}(x - 2cos3t)$

$xsin3t - ycos2t = 2sin3tcos3t - 3sin2tcos2t$

11.Find the equation of tangent and normal at any point to the curve given by $x = 7cost$ and $y = 2sint, \; t\epsilon R$

Soln: $x = 7cost \quad y = 2sint$

$m = \frac{dy}{dx} = -\frac{2\,cost \times dt}{7sint \times dt} = -\frac{2cost}{7sin}$

Equation of tangent $(y - y_1) = m(x - x_1)$

$y - 2sint = -\frac{2cos}{7sint}(x - 7cost)$

$x2cost + y7sint = 14(cos^2t + sin^2t) = 14$

Equation of normal $(y - y_1) = -\frac{1}{m}(x - x_1)$

$y - 2sint = \frac{7sint}{2c}(x - 7cost)$

$x7sint - y2cost = -4sintcost + 49sintcost$

12. If the curves $ax^2 + by^2 = 1$ and $cx^2 + dy^2 = 1$ intersect each other orthogonally then, $\frac{1}{a} - \frac{1}{b} = \frac{1}{c} - \frac{1}{d}$

Soln: Given $ax^2 + by^2 = 1$ and $cx^2 + dy^2 = 1$

$ax^2 + by^2 = cx^2 + dy^2$

$(a - c)x^2 = (d - b)y^2$

$\frac{x^2}{y^2} = \frac{d-b}{a-c}$……..(1)

$2ax + 2by\frac{dy}{dx} = 0 \qquad 2cx + 2dy\frac{dy}{dx} = 0$

$2by\frac{dy}{dx} = -2ax \qquad 2dy\frac{dy}{dx} = -2cx$

$m_1 = \frac{dy}{dx} = \frac{-a}{by} \qquad m_2 = \frac{dy}{dx} = \frac{-cx}{dy}$

Two curves intersect orthogonally then, $m_1\,m_2 = -1$

$\frac{acx^2}{bd\;^2} = -1 \qquad \frac{x^2}{y^2} = \frac{-bd}{ac}$………(2)

From 1 & 2 $\quad \frac{d-b}{a-c} = \frac{-b}{ac}$

$\frac{1}{a} - \frac{1}{b} = \frac{1}{c} - \frac{1}{d}$

13. Prove that the ellipse $x^2 + 4y^2 = 8$ and the hyperbola $x^2 - 2y^2 = 4$ intersect orthogonally. M-23

Soln: Given, $\quad x^2 + 4y^2 = 8 \quad x^2 - 2y^2 = 4$

Condition $\quad \frac{1}{a} - \frac{1}{b} = \frac{1}{c} - \frac{1}{d}$

$\frac{x^2}{8} + \frac{y^2}{2} = 1 \qquad \Rightarrow a = \frac{1}{8}, b = \frac{1}{2}$

$\frac{x^2}{4} + \frac{y^2}{-2} = 1 \qquad \Rightarrow c = \frac{1}{4}, d = \frac{-1}{2}$

L.H.S $\quad \frac{1}{a} - \frac{1}{b} = 8 - 2 = 6$

R.H.S $\quad \frac{1}{c} - \frac{1}{d} = 4 - (-2) = 6$

∴ the given curves intersect orthogonally.

14. Show that the two curves $x^2 - y^2 = r^2$ and $xy = c^2$ where c, r are constants, cut orthogonally.

Soln: $\quad x^2 - y^2 = r^2, \qquad xy = c^2$

$m_1 = 2x - 2y\frac{dy}{dx} = 0, \qquad m_2 = x\frac{dy}{dx} + y = 0$

$m_1 = \frac{dy}{dx} = \frac{-2x}{-2y} = \frac{x}{y}, \qquad m_2 = \frac{dy}{dx} = \frac{-y}{x}$

$m_1\,m_2 = -1$, ∴ the given curves intersect orthogonally.

15. We have a 12 square unit piece of thin material and want to make an open box by cutting small squares from the corners of our material and folding the sides up. The question is, which cut produces the box of maximum volume

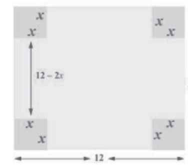
Soln: $\quad V = x(12 - 2x)^2$

$= x(144 - 48x + 4x^2)$

$V = 144x - 48x^2 + 4x^3$

$V' = 144 - 96x + 12x^2$

$V'' = -96 + 24x$



Put $V' = 0 \qquad 144 - 96x + 12x^2 = 0$

$x^2 - 8x + 12 = 0$

$\Rightarrow x = 2, 6$

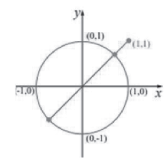
At $x = 2 \quad V'' < 0$

∴ Volume is maximum at $x = 2$

Required volume $V = 2(12 - 2 \times 2)^2 = 2(8)^2 = 128$

16. Find the points on unit circle $x^2 + y^2 = 1$ nearest and farthest point from (1,1)

Soln: Given $x^2 + y^2 = 1$



$2x + 2y\frac{dy}{dx} = 0 \quad \Rightarrow \frac{dy}{dx} = \frac{-x}{y}$

Distance between (x , y) and (1,1) is

$D = d^2 = (x - 1)^2 + (y - 1)^2$

$D' = 2(x - 1) + 2(y - 1)\frac{dy}{dx}$

$D' = \frac{2(x-y)}{y} \quad ; \quad D'' = \frac{2(x^2+y^2)}{y^3}$

Put $D' = 0 \Rightarrow \frac{2(x-y)}{y} = 0 \quad \Rightarrow x = y$

(1) $\Rightarrow 2x^2 = 1 \quad \Rightarrow x = \pm\frac{1}{\sqrt{2}} \quad y = \pm\frac{1}{\sqrt{2}}$

Required points $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ and $\left(\frac{-1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$

At $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) \quad D'' > 0 \quad$ ∴ the nearest point is $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

At $\left(\frac{-1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right) \quad D'' < 0 \quad$ ∴ the farthest point is $\left(\frac{-1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$

17. A steel plant is capable of producing x tonnes per day of a low-grade steel and y tonnes per day of a high-grade steel, where $y = \frac{40-5x}{10-x}$. If the fixed market price of low-grade steel is half that of high-grade steel, then what should be optimal productions in low-grade steel and high-grade steel in order to have maximum receipts.

Soln: Let the cost of low grade steel per tonne is Rs. P,

Let the cost of high grade steel per tonne is Rs. 2P,

Given $y = \frac{40-5x}{10-x}$.

Resultant cost $R = Px + 2Py$

$= Px + 2P\left(\frac{40-5x}{10-x}\right)$

$= P\left(\frac{80-x^2}{10-x}\right)$

d.w.r.t ‘x’ $\quad R' = P\left[\frac{x^2-20x+}{(10-x)^2}\right]$

$R'' = \frac{-40P}{(10-x)^3}$

$R' = 0 \quad \Rightarrow \quad x^2 - 20x + 80 = 0 \quad \Rightarrow \quad x = 10 \pm 2\sqrt{5}$

At $= 10 - 2\sqrt{5}, \quad R'' < 0$

$x = 10 - 2\sqrt{5}$ then $y = \frac{40-5(10-2\sqrt{5})}{10\;\;(10-2\sqrt{5})} = 5 - \sqrt{5}$

18. P.T all the rectangles of the given area square has the least perimeter.

Soln: Given $xy = A \Rightarrow y = \frac{A}{x}$

Perimeter $\quad$ P= $2(x + y)$

$= 2\left(x + \frac{A}{x}\right)$

d.w.r.t ‘x’ $\quad P' = 2\left(1 - \frac{A}{x^2}\right)$

$P'' = \frac{4A}{x^3} > 0$

put $P' = 0 \Rightarrow 1 - \frac{A}{x^2} = 0$

$x^2 = A$

$x = \sqrt{A}$

At $x = \sqrt{A} \quad P'' > 0$

$\therefore$ Perimeter is minimum at $x = \sqrt{A}$

$\therefore$ required numbers $x = \sqrt{A}$, $y = \frac{A}{\sqrt{A}} = \sqrt{A}$.

19.Find two positive numbers whose sum is 12 and their product is maximum. J-23, M-24

Soln: Given $x + y = 12 \Longrightarrow y = 12 - x$

Product $P = xy$

$= x(12 - x)$

$= 12x - x^2$

d.w.r.t 'x' $P' = 12 - 2x$

$P'' = -2 < 0$

put $P' = 0 \Longrightarrow 12 - 2x = 0$

$x = 6$

$P'' = -2 < 0$ $\therefore$ Product is maximum at $x = 6$

$\therefore$ required numbers 6 , 6.

20.Find two positive numbers whose product is 20 and their sum is minimum. S-21

Soln: Given $xy = 20 \Longrightarrow y = \frac{20}{x}$

Sum S= $x + y$

$= x + \frac{20}{x}$

d.w.r.t 'x' $S' = 1 - \frac{20}{x^2}$

$S'' = \frac{40}{x^3} > 0$

put $S' = 0 \Longrightarrow 1 - \frac{20}{x^2} = 0$

$x^2 = 20$

$x = 2\sqrt{5}$

At $x = 2\sqrt{5}$ $S'' > 0$

$\therefore$ Sum is minimum at $x = 2\sqrt{5}$

$\therefore$ required numbers $x = 2\sqrt{5}$, $y = \frac{20}{2\sqrt{5}} = 2\sqrt{5}$.

21.Find the smallest possible value of $x^2 + y^2$ given that $x + y = 10$.

Soln: Given $x + y = 10 \Longrightarrow y = 10 - x$

$f = x^2 + y^2$

$= x^2 + (10 - x)^2$

$= 2x^2 - 20x + 100$

d.w.r.t 'x' $f' = 4x - 20$

$f'' = 4 > 0$

put $f' = 0 \Longrightarrow 4x - 20 = 0$

$x = 5$

At $x = 5$ $f'' > 0$ $\therefore f$ is minimum at $x = 5$

$\therefore$ required numbers 5 , 5 .

22.A garden is to be laid out in a rectangular area and protected by wire fence. What is the largest possible area of the fenced garden with 40 metres of wire.

Soln: Perimeter $2(x + y) = 40$

$x + y = 20$

$y = 20 - x$

Area $A = xy$

$= x(20 - x)$

$= 20x - x^2$

d.w.r.t 'x' $A' = 20 - 2x$

$A'' = -2 < 0$

put $A' = 0 \Longrightarrow 20 - 2x = 0$

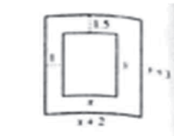
$x = 10$

$A'' = -2 < 0$ Area is maximum at $x = 10$

$\therefore$ $x = 10,$ $y = 20 - 10 = 10$.

$A = 10(10) = 100$ $\therefore$ largest possible area is 100.

23. A rectangular page is to contain 24 cm^2 of print. The margins at the top and bottom of the page are 1.5 cm and the margins at other sides of the page is 1 cm. What should be the dimensions of the page so that the area of the paper used is minimum.



Soln: printed area $A = xy = 24$

$y = \frac{24}{x}$

Length and breath of the paper $x + 2, y + 3$

Area of paper A= $(x + 2)(y + 3)$

$= (x + 2)\left(\frac{24}{x} + 3\right)$

$= 24 + 3x + 6 + \frac{48}{x}$

d.w.r.t 'x' $A' = 3 - \frac{48}{x^2}$

$A'' = \frac{96}{x^3} > 0$

put $A' = 0 \Longrightarrow 3 - \frac{48}{x^2} = 0$

$x^2 = 16 \Longrightarrow x = 4$

$A'' > 0$ $\therefore$ Area is minimum at $x = 4$ $y = 6$

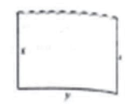
$\therefore$ Required length and breath

$x + 2 = 6$, $y + 3 = 6 + 3 = 9$.

24.A farmer plans to fence a rectangular pasture adjacent to a river. The pasture must contain 1,80,000 sq.mtrs in order to provide enough grass for herds. No fencing is needed along the river. What is the length of the minimum needed fencing material?

Soln: Given $A = xy = 180000$

$y = \frac{180000}{x}$



Perimeter P= $2x + y$

$= 2x + \frac{180000}{x}$

d.w.r.t 'x' $P' = 2 - \frac{180000}{x^2}$

$P'' = \frac{360000}{x^3} > 0$

put $P' = 0 \Longrightarrow 2 - \frac{180000}{x^2} = 0$

$x^2 = 90000 \Longrightarrow x = 300$

$P'' > 0$ $\therefore$ Perimeter is minimum at $x = 300$

$y = \frac{180000}{300} = 600$

$\therefore$ required material $2x + y = 2(300) + 600 = 1200$.

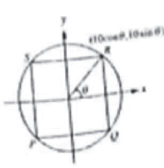
25. Find the dimensions of the rectangle with maximum area that can be inscribed in a circle of radius 10 cm.

Soln: Given $r = 10;$ $x = 10cos\theta,$ $y = 10sin\theta$

Length $2x = 20cos\theta,$ breadth $2y = 20sin\theta$

Area A= $400cos\theta sin\theta$

$= 200sin2\theta$



d.w.r.t 'x' $A' = 400cos2\theta$

$A'' = -800sin2\theta < 0$

put $A' = 0 \Longrightarrow 400cos2\theta = 0$

$cos2\theta = 0 \Longrightarrow \theta = \frac{\pi}{4}$

At $\theta = \frac{\pi}{4},$ $A'' < 0$

$\therefore$ Area is maximum at $\theta = \frac{\pi}{4}$

Length $2x = 20cos\theta = 20cos\frac{\pi}{4} = \frac{20}{\sqrt{2}},$

Breadth $2y = 20sin\theta = 20sin\frac{\pi}{4} = \frac{20}{\sqrt{2}}$

26. Prove that among all the rectangles of the given perimeter, the square has the maximum area.J-22,S-20

Soln: perimeter $P = 2(x + y)$

$y = \frac{P}{2} - x$

Area $A = xy$

$= x\left(\frac{P}{2} - x\right)$

$= \frac{Px}{2} - x^2$

d.w.r.t 'x' $A' = \frac{P}{2} - 2x$

$A'' = -2 < 0$

put $A' = 0 \Longrightarrow \frac{P}{2} - 2x = 0$

$x = \frac{P}{4}$

$A'' < 0$ Area is maximum at $x = \frac{P}{4}$

$\therefore$ $x = \frac{P}{4},$ $y = \frac{P}{2} - x = \frac{P}{2} - \frac{P}{4} = \frac{P}{4}$.

Among all the rectangles of the given perimeter, the square has the maximum area.

27. Find the dimensions of the largest rectangle that can be inscribed in a semi circle of radius *r* cm.

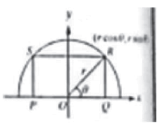
Soln: Given $r = 10$; $x = rcos\theta$, $y = rsin\theta$

Length $2x = 2rcos\theta$, breadth $y = rsin\theta$

Area A= $2xy$

$= 2r^2cos\theta sin\theta$

$= r^2 sin2\theta$



d.w.r.t 'x' $A' = 2r^2cos2\theta$

$A'' = -4r^2sin2\theta < 0$

put $A' = 0 \Longrightarrow 2r^2cos2\theta = 0$

$cos2\theta = 0 \Longrightarrow \theta = \frac{\pi}{4}$

At $\theta = \frac{\pi}{4}$, $A'' < 0$ ∴ Area is maximum at $\theta = \frac{\pi}{4}$

Length $2x = 2rcos\theta = 2rcos\frac{\pi}{4} = \frac{2r}{\sqrt{2}}$,

Breadth $y = rsin\theta = rsin\frac{\pi}{4} = \frac{r}{\sqrt{2}}$

28. A manufacturer wants to design an open box having a square base and a surface area of 108 sq.cm. Determine the dimensions of the box for the maximum volume.

Soln: Dimensions of box are x,x,y

Surface area $S = x^2 + 4xy = 108$

$4xy = 108 - x^2$

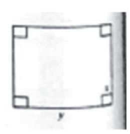
$y = \frac{108}{4x} - \frac{x^2}{4x}$

$y = \frac{27}{x} - \frac{x}{4}$

Volume $V = x^2y$

$= x^2\left(\frac{27}{x} - \frac{x}{4}\right)$

$= 27x - \frac{x^3}{4}$



d.w.r.t 'x' $V' = 27 - \frac{3x^2}{4}$

$V'' = \frac{-6x}{4}$

put $V' = 0 \Longrightarrow 27 - \frac{3x^2}{4} = 0$

$x^2 = 36 \Longrightarrow x = 6$

Dimensions of box are $x = 6$, $x = 6$, $y = \frac{27}{6} - \frac{6}{4} = 3$.

29. The volume of a cylinder is given by the formula $\pi r^2 h$. Find greatest and least values of *V* if $r + h = 6$.

Soln: Given $r + h = 6 \Longrightarrow h = 6 - r$

Volume $V = \pi r^2 h$

$= \pi r^2(6 - r)$

$V = 6\pi r^2 - \pi r^3$

d.w.r.t 'x' $V' = 12\pi r - 3\pi r^2$

$V'' = 12\pi - 6\pi r$

$V' = 0 \Longrightarrow 12\pi r - 3\pi r^2 = 0$

$r = 0$, $r = 4$

At $r = 0$ $V'' > 0$ ∴ V is minimum, $V = 0$

At $r = 4$ $V'' < 0$ ∴ V is maximum, $V = 32\pi$

30. A hollow cone with base radius *a* cm and height *b* cm is placed on a table. Show that the volume of the largest cylinder that can be hidden underneath is $\frac{4}{9}$ times volume of the cone.

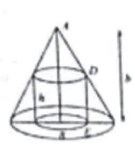
Soln: From similar triangle $\frac{b-h}{r} = \frac{b}{a} \Longrightarrow \frac{a(b-h)}{b} = r$

Volume of cylinder V= $\pi r^2 h = \pi\left(\frac{a(b-h)}{b}\right)^2 h$

$V = \frac{\pi a^2}{b^2}(b^2h + h^3 - 2bh^2)$

d.w.r.t 'h' $V' = \frac{\pi a^2}{b^2}(b^2 + 3h^2 - 4bh)$

$V'' = \frac{\pi a^2}{b^2}(6h - 4b)$



put $V' = 0 \Longrightarrow b^2 - 4bh + 3h^2 = 0$

$h = \frac{b}{3}$ or $h = b$(not possible)

$V'' < 0$ ∴ volume maximum at $h = \frac{b}{3}$

Volume of cylinder V = $\frac{\pi a^2}{b^2}\left(\frac{b^3}{3} + \frac{b^3}{27} - \frac{2b^3}{9}\right)$

$= \frac{4}{27}\pi a^2 b = \frac{4}{9}\left(\frac{1}{3}\pi a^2 b\right)$

$= \frac{4}{9}$(Volume of cone)

31. If $y = x^2 - 5x + 4$ whose tangent is parallel to $3x + y = 7$ show that the point of contact is (1,0).

Soln: $y = x^2 - 5x + 4$ $3x + y = 7$ S-21

$m_1 = \frac{dy}{dx} = 2x - 5$ $y = -3x + 7$ $m_2 = -3$

Tangent is parallel ∴ $m_1 = m_2$

$2x - 5 = -3$

$2x = 2$

$x = 1$

$x = 1 \Longrightarrow y = 1^2 - 5(1) + 4 = 0$

The point of contact is (1,0)

32. Find point on $y = x^3 - 3x^2 + x - 2$ at which tangent is parallel to $y = x$. M-22

Soln: $y = x^3 - 3x^2 + x - 2$ $y = x$

$m_1 = \frac{dy}{dx} = 3x^2 - 6x + 1$ $m_2 = 1$

Tangent is parallel ∴ $m_1 = m_2$

$3x^2 - 6x + 1 = 1$

$3x(x - 2) = 0 \Longrightarrow x = 0, x = 2$

$x = 0 \Longrightarrow y = 0^3 - 3(0)^2 + 0 - 2 = -2$

$x = 2 \Longrightarrow y = 2^3 - 3(2)^2 + 2 - 2 = -4$

the point of contact is (0,-2)and (2,-4).

33. Find the equation of tangent and normal to the curve $y = x^2 - x^4$ at (1,0). J-22

Soln: $y = x^2 - x^4$

$m = \frac{dy}{dx} = 2x - 4x^3$

At (1,0) $\Longrightarrow$ $m = 2(1) - 4(1)^3 = -2$

Equation of tangent line $y - y_1 = m(x - x_1)$

$y - 0 = -2(x - 1)$

$y = -2x + 2 \Longrightarrow 2x + y - 2 = 0$

Equation of normal line $y - y_1 = \frac{-1}{m}(x - x_1)$

$y - 0 = \frac{-1}{-2}(x - 1)$

$2y = x - 1$ or $x - 2y - 1 = 0$.

34. Find the equation of tangent and normal to the curve $y = x^2 + 3x - 2$ at (1,2). M-23

Soln: $y = x^2 + 3x - 2$ $\Longrightarrow$ $m = \frac{dy}{dx} = 2x + 3$

At (1,2) $\Longrightarrow$ $m = 2(1) + 3 = 5$

Equation of tangent line $y - y_1 = m(x - x_1)$

$y - 2 = 5(x - 1)$

$y - 2 = 5x - 5$

$5x - y - 3 = 0$

Equation of normal line $y - y_1 = \frac{-1}{m}(x - x_1)$

$y - 2 = \frac{-1}{5}(x - 1)$

$5y - 10 = -x + 1$

$x + 5y - 11 = 0.$

35. Find the absolute maximum and absolute minimum of the function $f(x) = 2x^3 + 3x^2 - 12x$ on $[-3, 2]$

Soln: $f(x) = 2x^3 + 3x^2 - 12x$

$f'(x) = 6x^2 + 6x - 12$

$f'(x) = 0 \Longrightarrow 6x^2 + 6x - 12 = 0$

$x^2 + x - 2 = 0$

$x = -2, x = 1$

$f(-3) = 2(-3)^3 + 3(-3)^2 - 12(-3) = -54 + 27 + 36 = 9$

$f(2) = 2(2)^3 + 3(2)^2 - 12(2) = 16 + 12 - 24 = 4$

$f(-2) = 2(-2)^3 + 3(-2)^2 - 12(-2) = -16 + 12 + 24 = 20$

$f(1) = 2(1)^3 + 3(1)^2 - 12(1) = 2 + 3 - 12 = -7$

Absolute maximum of f is 20 at $x = -2$

Absolute minimum of f is -7 at $x = 1$.

36. Find the intervals of monotonicity and hence find the local extrema for the fun $f(x) = x^2 - 4x + 4$

Soln:

$f(x) = x^2 - 4x + 4$

$f'(x) = 2x - 4$
$f'(x) = 0 \quad \Rightarrow \quad 2x - 4 = 0$
$x = 2$
$f'(x) > 0$ $f(x)$ is strictly increasing in $(2, \infty)$
$f'(x) < 0$ $f(x)$ is strictly decreasing in $(-\infty, 2)$
$f'(x)$ sign changes negative to positive at $x = 2$
$f(x)$ attains local minimum at $x = 2$
Local minimum of $f(x)$ is $f(2) = 0$.

37.Determine the intervals of concavity of the curve $f(x) = (x-1)^3(x-5)$ and point of inflections.

Soln: $f(x) = (x-1)^3(x-5)$
$f'(x) = (x-1)^3 + (x-5)3(x-1)^2$
$f'(x) = 4(x-1)^2(x-4)$
$f''(x) = 4[(x-1)^2 + 2(x-1)(x-4)]$
$= 12(x-1)(x-3)$
$f''(x) = 0 \quad \Rightarrow \quad 12(x-1)(x-3) = 0$
$x = 1, \ x = 3$

Intervall	$f''(x)$	concavity
$(-\infty, 1)$	+	Upward
$(1, 3)$	-	Downward
$(3, \infty)$	+	upward

Point of inflection is
$(1, f(1)) = (1, 0)$ and $(3, f(3)) = (3, -16)$

38.Find the local extrema of the fun $f(x) = x^4 + 32x$

Soln: $f(x) = x^4 + 32x$
$f'(x) = 4x^3 + 32$
$f''(x) = 12x^2$
$f'(x) = 0 \quad \Rightarrow \quad 4x^3 + 32 = 0$
$x = -2$
$f''(-2) > 0$ $f(x)$ attains local minimum at $x = -2$
Local minimum of $f(x)$ is $f(-2) = -48$
Local extrema of $f(x)$ is $(-2, -48)$)

39.Find the local extrema of the fun $f(x) = 4x^6 - 6x^4$ M-22

Soln: $f(x) = 4x^6 - 6x^4$
$f'(x) = 24x^5 - 24x^3$
$f''(x) = 120x^4 - 72x^2$
$f'(x) = 0 \Rightarrow 24x^3(x^2 - 1) = 0$
$x = 0, \quad x = 1, \quad x = -1$
$f''(0) = 0$ second derivative test fails

Intervall	$f'(x)$	Monotonicity
$(-\infty, -1)$	-	Strictly decreasing
$(-1, 0)$	+	Strictly increasing
$(0, 1)$	-	Strictly decreasing
$(1, \infty)$	+	Strictly increasing

$f(x)$ attains local minimum at $x = -1$,
local minimum value of $f(-1) = -2$
$f(x)$ attains local maximum at $x = 0$,
local maximum value of $f(0) = 0$
$f(x)$ attains local minimum at $x = 1$,
local minimum value of $f(1) = -2$.

9.Applications of Integration

1 Evaluate $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \boldsymbol{x\cos x\, dx}$

Solu: $f(x) = x\cos x$
$f(-x) = -f(x)$ $f(x)$ ஓர் ஒற்றை சார்பு
$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x\cos x\, dx = 0.$

2. Evaluate $\int_{-5}^{5} \frac{e^x - 1}{e^x + 1} dx$

Solu: $f(x) = \frac{e^x - 1}{e^x + 1}$
$f(-x) = -f(x)$ $f(x)$ ஓர் ஒற்றை சார்பு
$\int_{-5}^{5} \frac{e^x - 1}{e^x + 1} dx = 0.$

3. Evaluate $\int_{-5}^{5} x\cos\left(\frac{e^x - 1}{e^x + 1}\right) dx$

Solu: $f(x) = x\cos\left(\frac{e^x - 1}{e^x + 1}\right)$
$f(-x) = -f(x)$ $f(x)$ *is odd function.*
$\int_{-5}^{5} x\cos\left(\frac{e^x - 1}{e^x + 1}\right) dx = 0.$

4. Evaluate $\int_{-log2}^{log2} e^{-|x|} dx$

Solu: $f(x) = e^{-|x|}$ $f(x)$ *is even function*

$$\int_{-log2}^{log2} e^{-|x|} dx = 2\int_{0}^{log2} e^{-x} dx$$
$$= 2(-e^{-x})_0^{log2}$$
$$= -2(e^{-log2} - e^{-0})$$
$$= -2\left(\frac{1}{2} - 1\right)$$
$$= 1$$

5. Evaluate $\int_0^1 \frac{\sqrt{x}}{\sqrt{x} + \sqrt{1-x}} dx$

Solu: $I = \int_0^1 \frac{\sqrt{x}}{\sqrt{x} + \sqrt{1-x}} dx$ ....*(1)*

$I = \int_0^1 \frac{\sqrt{1-x}}{\sqrt{1-x} + \sqrt{x}} dx$ ....*(2)*

(1)+(2) $\Rightarrow$ $2I = \int_0^1 \frac{\sqrt{x} + \sqrt{1-x}}{\sqrt{1-x} + \sqrt{x}} dx$

$$= \int_0^1 (1)dx$$
$$2I = (x)_0^1 = 1$$
$$\Rightarrow I = \frac{1}{2}$$

$$\int_0^1 \frac{\sqrt{x}}{\sqrt{x} + \sqrt{1-x}} dx = \frac{1}{2}$$

6. Evaluate $\int_2^3 \frac{\sqrt{x}}{\sqrt{x} + \sqrt{5-x}} dx$

Solu: $I = \int_2^3 \frac{\sqrt{x}}{\sqrt{x} + \sqrt{5-x}} dx$ ....*(1)*

$I = \int_2^3 \frac{\sqrt{5-x}}{\sqrt{5-x} + \sqrt{x}} dx$ ....*(2)*

(1)+(2) $\Rightarrow$ $2I = \int_2^3 \frac{\sqrt{5-x} + \sqrt{x}}{\sqrt{x} + \sqrt{5-x}} dx$

MATHS

$= \int_2^3 (1)\,dx$

$= (x)_2^3$

$= 3 - 2$

$= 1 \implies I = \frac{1}{2}$

$$\int_2^3 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{5-x}}\,dx = \frac{1}{2}$$

7. Evaluate $\int_0^a \frac{f(x)}{f(x)+f(a-x)}\,dx$ *M-22*

Solu: $I = \int_0^a \frac{f(x)}{f(x)+f(a-x)}\,dx$ ………(1)

$I = \int_0^a \frac{f(a-x)}{f(a-x)+f(x)}\,dx$ ……….(2)

(1)+(2) $\implies 2I = \int_0^a \frac{f(x)+f(a-x)}{f(x)+f(a-x)}\,dx$

$2I = \int_0^a (1)\,dx$

$2I = (x)_0^a$

$\implies 2I = a - 0 \implies I = \frac{a}{2}$

$$\int_0^a \frac{f(x)}{f(x)+f(a-x)}\,dx = \frac{a}{2}$$

8. Evaluate $\int_0^{\frac{\pi}{2}} \frac{f(sinx)}{f(sinx)+f(cosx)}\,dx$ *M-20*

Solu: $I = \int_0^{\frac{\pi}{2}} \frac{f(sinx)}{f(sinx)+f(cosx)}\,dx$ ………(1)

$I = \int_0^{\frac{\pi}{2}} \frac{f(cosx)}{f(cosx)+f(sinx)}\,dx$ ……….(2)

(1)+(2) $\implies 2I = \int_0^{\frac{\pi}{2}} \frac{f(sinx)+f(cosx)}{f(sinx)+f(cosx)}\,dx$

$2I = \int_0^{\frac{\pi}{2}} (1)\,dx$

$2I = (x)_0^{\frac{\pi}{2}} \implies 2I = \frac{\pi}{2} - 0 \implies I = \frac{\pi}{4}$

$$\int_0^{\frac{\pi}{2}} \frac{f(sinx)}{f(sinx)+f(cosx)}\,dx = \frac{\pi}{4}$$

9 Evaluate $\int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \frac{1}{1+\sqrt{tanx}}\,dx$ *M-24*

Solu:

$I = \int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \frac{1}{1+\sqrt{\frac{sinx}{cosx}}}\,dx = \int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \frac{\sqrt{cosx}}{\sqrt{cosx}+\sqrt{sinx}}\,dx$ ……(1)

$I = \int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \frac{\sqrt{sinx}}{\sqrt{cosx}+\sqrt{sinx}}\,dx$ ……….(2)

(1)+(2) $\implies 2I = \int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \frac{\sqrt{cosx}+\sqrt{sinx}}{\sqrt{cosx}+\sqrt{sinx}}\,dx$

$2I = \int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} (1)\,dx$

$2I = (x)_{\frac{\pi}{8}}^{\frac{3\pi}{8}}$ $\quad 2I = \frac{3\pi}{8} - \frac{\pi}{8}$

$2I = \frac{2\pi}{8} \implies I = \frac{\pi}{8}$

10 Evaluate $\int_0^1 |5x-3|\,dx$

Solu:

$\int_{-4}^{4} |x+3|\,dx = \int_0^{\frac{3}{5}} (-5x+3)\,dx + \int_{\frac{3}{5}}^{1} (5x-3)\,dx$

$= [-\frac{5x^2}{2} + 3x]_0^{\frac{3}{5}} + [\frac{5x^2}{2} - 3x]_{\frac{3}{5}}^{1}$

$= \left[\left(-\frac{9}{10}+\frac{9}{5}\right) - (0)\right] + \left[\left(\frac{5}{2}-3\right) - \left(\frac{9}{10}-\frac{9}{5}\right)\right]$

$= \frac{9}{5} - \frac{1}{2} + \frac{9}{5}$

$= \frac{9}{5} - \frac{1}{2}$

$= \frac{18-5}{10}$

$= \frac{13}{10}$

11 Evaluate $\int_{-4}^{4} |x+3|\,dx$

Solu:

$\int_{-4}^{4} |x+3|\,dx = \int_{-4}^{-3} (-x-3)\,dx + \int_{-3}^{4} (x+3)\,dx$

$= [-\frac{x^2}{2} - 3x]_{-4}^{-3} + [\frac{x^2}{2} + 3x]_{-3}^{4}$

$= \left[\left(-\frac{9}{2}+9\right) - \left(-\frac{16}{2}+12\right)\right] + \left[\left(\frac{16}{2}+12\right) - \left(\frac{9}{2}-9\right)\right]$

$= \frac{9}{2} - 4 + 20 + \frac{9}{2}$

$= 25$

12. Evaluate

(i) $\int xe^x dx$

Solu: $\int xe^x dx = xe^x - e^x + c$

$= e^x(x-1) + c$

(ii) $\int_0^1 xe^x\ dx = 1$

$\int_0^1 xe^x\ dx = [e^x(x-1)]_0^1$

$= 0 - (-1) = 1$

13. Evaluate $\int x^3 e^x dx$

Solu:

$\int x^3 e^x dx = x^3e^x - 3x^2e^x + 6xe^x - 6e^x + c$

$= e^x(x^3 - 3x^2 + 6x - 6) + c.$

14 *Evaluate* $\int_0^{\frac{\pi}{2}} \sin^{10} x\, dx$ *M-24*

Solu:

$$\int_0^{\frac{\pi}{2}} \sin^{10}x\, dx = \frac{9\times7\times5\times3\times1}{10\times8\times6\times4\times2}\times\frac{\pi}{2} = \frac{63\pi}{512}$$

15. *Evaluate* $\int_0^{\frac{\pi}{2}} \sin^9 x\, dx$

Solu:

$$\int_0^{\frac{\pi}{2}} \sin^9 x\, dx = \frac{8\times6\times4\times2}{9\times7\times5\times3} = \frac{128}{315}$$

16. *Evaluate* $\int_0^{\frac{\pi}{2}} \sin^6 x\cos^4 x\, dx$

Solu:

$$\int_0^{\frac{\pi}{2}} \sin^6 x\cos^4 x\, dx = \frac{5\times3\times1\times3\times1}{10\times8\times6\times4\times2}\times\frac{\pi}{2} = \frac{3\pi}{512}$$

17. *Evaluate* $\int_0^{\frac{\pi}{2}} \sin^5 x\cos^4 x\, dx$

Solu:

$$\int_0^{\frac{\pi}{2}} \sin^5 x\cos^4 x\, dx = \frac{4\times2\times3\times1}{9\times7\times5\times3} = \frac{8}{315}$$

18. *Evaluate* $\int_0^{\frac{\pi}{2}} \sin^3 x\cos^5 x\, dx$

Solu:

$$\int_0^{\frac{\pi}{2}} \sin^3 x\cos^5 x\, dx = \frac{2\times4\times2}{8\times6\times4\times2} = \frac{1}{24}$$

Solu:

19. *Evaluate* $\int_0^{2\pi} \sin^7 \frac{x}{4}\, dx$

Solu:

$$\int_0^{2\pi} \sin^7 \frac{x}{4}\, dx = 4\int_0^{\frac{\pi}{2}} \sin^7 x\, dx$$

$$= 4\left[\frac{6\times4\times2}{7\times5\times3}\right] = \frac{64}{35}$$

20. *Evaluate* $\int_0^{\frac{\pi}{4}} \sin^6 2x\, dx$

Solu:

$$\int_0^{\frac{\pi}{4}} \sin^6 2x\, dx = \frac{1}{2}\int_0^{\frac{\pi}{2}} \sin^6 x\, dx$$

$$= \frac{1}{2}\left[\frac{5\times3\times1}{6\times4\times2}\times\frac{\pi}{2}\right] = \frac{5\pi}{64}$$

21. *Evaluate* $\int_0^1 x^3 (1-x)^4 dx$

Solu:

$$\int_0^1 x^3 (1-x)^4 dx = \frac{3!\times4!}{(3+4+1)!} = \frac{3!\times4!}{8!} = \frac{1}{280}$$

22. *Evaluate* $\int_0^1 x^5 (1-x^2)^5 dx$

Solu:

$$\int_0^1 x^5 (1-x^2)^5 dx = \frac{1}{2}\int_0^1 (x^2)^2 (1-x^2)^5 d(x^2)$$

$$= \frac{1}{2}\left[\frac{2!\times5!}{8!}\right] = \frac{1}{336}$$

23. *Evaluate* $\int_0^{\infty} x^5 e^{-3x} dx$ *J-23*

Solu:

$$\int_0^{\infty} x^5 e^{-3}\ dx = \frac{5!}{3^{5+1}} = \frac{5!}{3^6}$$

24. *Evaluate* $\int_b^{\infty} \frac{1}{a^2+x^2} dx$ *M-23*

Solu:

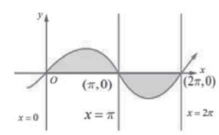
$$\int_b^{\infty} \frac{1}{a^2+x^2} dx = \left[\frac{1}{a}\tan^{-1}\frac{x}{a}\right]_b^{\infty}$$

$$= \left[\frac{1}{a}\tan^{-1}\frac{\infty}{a}\right] - \left[\frac{1}{a}\tan^{-1}\frac{b}{a}\right]$$

$$= \frac{1}{a}\left[\frac{\pi}{2} - \tan^{-1}\frac{b}{a}\right]$$

25. $y = \sin x$ என்ற வளைவரை, x -அச்சு, கோடுகள் $x = 0$ மற்றும் $x = 2\pi$ ஆகியவற்றால் அடைபடும் அரங்கத்தின் பரப்பைக் காண்க.

Solu:



$$A = \int_0^{2\pi} y\,dx$$

$$= 2\int_0^{\pi} y\,dx$$

$$= 2\int_0^{\pi} \sin x\,dx$$

$$= 2(-\cos x)_0^{\pi}$$

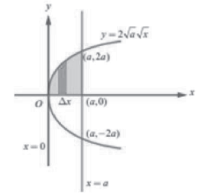
$$= 2(1+1)$$

$$= 4$$

26. $y^2 = 4ax$ என்ற பரவளையத்திற்கும் அதன் செவ்வகலத்திற்கும் அடைபடும் அரங்கத்தின் பரப்பைக் காண்க. *J-22*

Solu: $y^2 = 4ax \Rightarrow y = 2\sqrt{a}\sqrt{x}$

$A = 2\int_0^a y\,dx$



$$= 2\int_0^a 2\sqrt{a}\sqrt{x}\,dx$$

$$= 4\sqrt{a}\left(\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right)_0^a$$

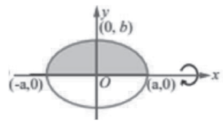
$$= \frac{8\sqrt{a}\times a^{\frac{3}{2}}}{3}$$

$$= \frac{8a^2}{3}$$

MATHS

27. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, a>b$ என்ற அடைபடும் அரங்கத்தின் பரப்பினை நெட்டச்சைப் பொருத்துச் சுழற்றினால் உருவாகும் திடப்பொருளின் கனஅளவைக் காண்க.

Solu: $V=\pi\int_{-a}^{a} y^2 dx$



$$=\pi\int_{-a}^{a}\frac{b^2}{a^2}(a^2-x^2)dx$$

$$=2\pi\frac{b^2}{a^2}\left[a^2x-\frac{x^3}{3}\right]_0^a$$

$$=2\pi\frac{b^2}{a^2}\left(a^3-\frac{a^3}{3}\right)$$

$$=2\pi\frac{b^2}{a^2}\left(\frac{2a^3}{3}\right)$$

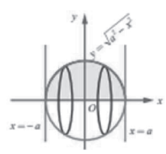
$$=\frac{4\pi ab^3}{3},$$

28. ஆரம் a உடைய கோளத்தின் கன அளவைக் காண்க.

Solu: $V=\pi\int_{-a}^{a} y^2 dx$

$$=\pi\int_{-a}^{a}(a^2-x^2)dx$$

$$=2\pi\left[a^2x-\frac{x^3}{3}\right]_0^a$$

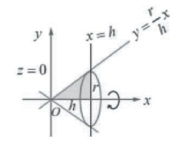


$$=2\pi\left(a^3-\frac{a^3}{3}\right)$$

$$=2\pi\left(\frac{2a^3}{3}\right)=\frac{4\pi a^3}{3}$$

29. ஆரம் r மற்றும் உயரம் h உடைய நேர்வட்டக் கூம்பின் கன அளவைக் காண்க.

Solu: $V=\pi\int_{-a}^{a} y^2 dx$



$$=\pi\int_0^h\left(\frac{r}{h}x\right)^2dx$$

$$=\pi\left(\frac{r}{h}\right)^2\left[\frac{x^3}{3}\right]_0^h$$

$$=\pi\frac{r^2}{h^2}\left(\frac{h^3}{3}\right)$$

$$=\pi\left(\frac{r^2h}{3}\right)$$

$$=\frac{1}{3}\pi r^2h$$

ANSWERS

CHAPTER -1

1.b	2.a	3.d	4.b	5.c	6.b	7.c	8.d	9.b	10.d
11.b	12.a	13.b	14.d	15.a	16.d	17.b	18.d	19.b	20.d
21.d	22.c	23.a	24.d	25.d					

CHAPTER -2

1.c	2.a	3.d	4.a	5.a	6.a	7.a	8.b	9.d	10.b
11.c	12.a	13.a	14.b	15.d	16.b	17.a	18.a	19.b	20.c
21.a	22.c	23.d	24.d	25.b					

CHAPTER -3

1.a	2.b	3.a	4.c	5.a	6.d	7.d	8.a	9.c	10.c

CHAPTER -4

1.a	2.b	3.c	4.b	5.a	6.c	7.a	8.c	9.c	10.b
11.b	12.d	13.d	14.d	15.d	16.c	17.b	18.b	19.a	20.c

CHAPTER -5

1.a	2.d	3.c.	4.c	5.a	6.d	7.d	8.a	9.a	10.b
11.b	12.c	13.c	14.c	15.b	16.a	17.c	18.d	19.c	20.c
21.a	22.a	23.c	24.b	25.b					

CHAPTER -6

1.a	2.a	3.b	4.c	5.a	6.c	7.d	8.c	9.a	10.b
11.a	12.c	13.a	14.a	15.b	16.d	17.d	18.b	19.c	20.d
21.b	22.a	23.b	24.c	25.d					

CHAPTER -7

1.d	2.b	3.c	4.c	5.a	6.d	7.c	8.a	9.b	10.b
11.b	12.a	13.a	14.d	15.c	16.c	17.d	18.b	19.c	20.c

CHAPTER -8

1.a	2.b	3.c	4.b	5.b	6.b	7.d	8.b	9.a	10.c
11.b	12.d	13.c	14.b	15.d					

CHAPTER -9

1.d	2.d	3.c	4.a	5.c	6.c	7.c	8.c	9.b	10.a
11.a	12.d	13.b	14.d	15.d	16.b	17.b	18.d	19.d	20.c

CHAPTER -10

1.a	2.a	3.b	4.b	5.a	6.a	7.b	8.c	9.b	10.b
11.c	12.a	13.b	14.c	15.b	16.d	17.b	18.c	19.b	20.b
21.c	22.c	23.c	24.a	25.d					

CHAPTER -11

1.d	2.b	3.a	4.b	5.d	6.b	7.c	8.d	9.d	10.d
11.b	12.d	13.d	14.b	15.a	16.a	17.b	18.a	19.a	20.d

CHAPTER -12

1.b	2.b	3.c	4.d	5.c	6.b	7.c	8.a	9.c	10.c
11.c	12.b	13.b	14.d	15.b	16.d	17.a	18.d	19.d	20.c

All the best